

Math 3322
Test V

For this test, you are allowed to consult with any individual, classmate or faculty member and use any book as reference. You will be graded on the mathematical correctness, written presentation and clarity of your solutions. Please put no more than one question per page of paper (multiple parts of the same question are ok), write only on one side and show all details. This test is due Dec. 10th before 10:45 AM.

1. (15 points) Determine, with proof, when the sequence of n 5's is graphical.
2. (15 points) Use induction to show $\frac{10^{2n+1} + 7^{n-3} + 22}{3}$ is an integer for all positive integers $n \geq 3$.
3. (15 points) Find a graph G such that $G \cong \overline{G}$
 - i. with $n = 4$ vertices.
 - ii. with $n = 5$ vertices.

Can you find a graph G with $n = 6$ vertices such that $G \cong \overline{G}$? If yes, provide an example of such a graph. If no, prove why it is not possible.

4. (15 points) Find the coefficient of x^6 in the polynomial expansion of $(2x^2 - 1)^9$. Find the coefficient of x^4 in the polynomial expansion of $(x + 2)^5(2x - 1)$. Find the sum of the coefficients in the expansion of $(x + 1)^{33}$ (You may leave this answer in exponential form).
5. (20 points) Two married couples, two single men and one single woman sit in a row of seven consecutive seats. How many ways can they be seated
 - i. with no restrictions;
 - ii. alternating genders;
 - iii. such that the women are all consecutive;
 - iv. such that spouses sit next to one another?
6. (20 points) Determine the number of different arrangements of the letters in the presidential names below. Keep in mind that you can distinguish between uppercase and lowercase letters.
 - i. *Ford*;
 - ii. *Kennedy*;
 - iii. *Lincoln*;
 - iv. *Eisenhower*.
7. (15 points) While playing poker, a player holds the five cards $2\clubsuit, 5\diamond, \text{King}\diamond, \text{King}\spadesuit$ and $\text{Ace}\clubsuit$ in her hand. If she discards the $2\clubsuit, 5\diamond$ and $\text{Ace}\clubsuit$ (without replacing them into the deck) and draws three more cards, what is the probability that the result will be a *three of a kind* or better?