Math 3322
Test IV

1. Complete the following.

(3 points) The graph K_{42} has____________edges.

(3 points) The graph N_{53} has ____________ edges.

(3 points) In graph theory, walk is to path as ____________ is to ____________.

(3 points) The graph K_{435} **is/is not** Eulerian.

(3 points) Draw an example of a connected graph with six vertices and five different vertex degrees.

2. (10 points) Suppose a graph has 15 edges, 3 vertices of degree 4, and all others of degree 3. How many vertices does the graph have?

3. (10 points) Let G be a connected graph on n vertices with a cutvertex v. When v is removed what is the minimum number of components that could be created in the disconnected graph? What is the maximum number? Use examples to illustrate the answers.

4. (15 points) State the definition of *regular of degree r*.

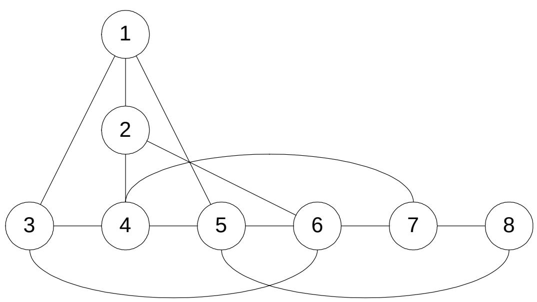
A. If a graph with n vertices is regular of degree r, how many edges does it have?

B. The graph K_n is regular of degree __________.

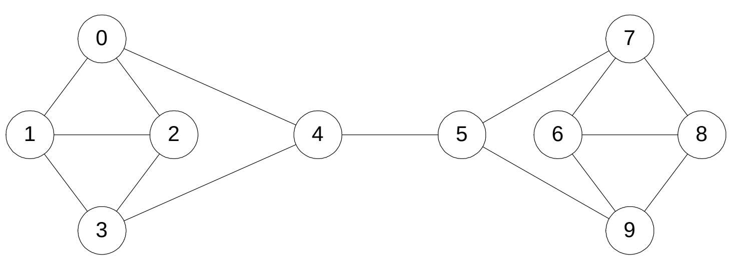
Using both **A** and **B**, K_n has _______________ edges.

5. (15 points) Determine if the following graphs are Hamiltonian. Be sure to justify your answers.

i.



ii.



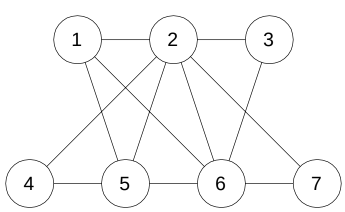
6. Consider the graph G with vertex set $V = \{2, 3, 4, ..., 98, 99, 100\}$ where the $i - j$ edge is in the edge set E if and only if i divides j for $i < j$ (i.e. $\frac{j}{i}$ is an integer).

i. (5 points) Prove that G is disconnected.

ii. (5 points) Find a subgraph K_5 in G.

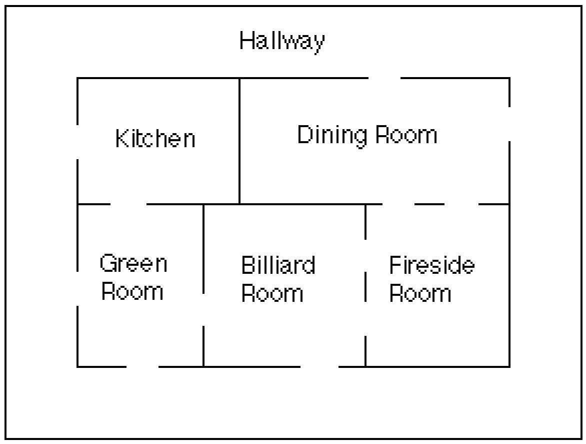
iii. (5 points) Prove that G is not Eulerian.

7. (10 points) Compute the independence number and clique number for the following graph. Be sure to indicate the appropriate subgraph used to compute these values and argue why your examples are the maximum subgraph possible.



8. (10 points) King Euler was a king who loved mathematics. He especially loved puzzles that involved mathematics. One day he was looking at a blueprint of one part of the castle and came up with the following problem.

In the blueprint shown below there are five rooms. Many of the rooms are connected by doorways. For example, there are two doorways between the Billiard Room and the Fireside Room, and one door connecting the Billiard Room and the Green Room. One long rectangular hallway goes around these five rooms.



Is it possible to start in the hallway, walk through every doorway once, and end in the hallway? Explain why or why not. Correct solutions found without using graph theory will be awarded no points.

Bonus! (10 points) True or False? An Eulerian graph contains no bridges. If true, prove it. If false, construct a counterexample.