Math 3322
Test III
DeMaio

1. (15 points) Use induction to prove $\frac{2^{3n}-22}{7}$ is an integer for all positive integers n.

2. (15 points) Use a combinatorial proof to show $\binom{3n}{3} = n^3 + 6n\binom{n}{2} + 3\binom{n}{3}$.

3. (10 points) Expand $(2x-3)^6$ into polynomial form.

4. (10 points) Find the coefficient of x^6 in the polynomial expansion of $(2x^2-1)^{13}$.

5. (5 points) Find the coefficient of x^7 in the polynomial expansion of $(2x^2-1)^{13}$.

6. (15 points) Simplify $\sum_{i=0}^{n}(-0.8)^{3i}2^{i}(-5)^{3n}\binom{n}{i}$.

Hint! Think of an appropriate substitution for –0.8

The French mathematician, Edouard Lucas (1842-1891), who gave the series of numbers 0, 1, 1, 2, 3, 5, 8, 13, .. the name *the Fibonacci Numbers*, found a similar series occurs often when investigating Fibonacci number patterns: 2, 1, 3, 4, 7, 11, 18, ...
The Fibonacci rule of adding the latest two to get the next is kept, but here we start from 2 and 1 (in this order) instead of 0 and 1 for the (ordinary) Fibonacci numbers. The series, called the **Lucas Numbers** after him, is defined as follows: $L_0 = 2$, $L_1 = 1$ and $L_n = L_{n-1} + L_{n-2}$ for $n \geq 2$ where we write its members as L_n, for Lucas.

n	0	1	2	3	4	5	6	7	8	9	10
F_n	0	1	1	2	3	5	8	13	21	34	55
L_n	2	1	3	4	7	11	18	29	47	76	123

7. (10 points) Use induction to prove $\sum_{i=0}^{n} (L_i)^2 = L_n L_{n+1} + 2$ for $n \geq 0$.

8. (5 points) Compute $L_{n-1} + L_{n+1}$ for

i. $n = 1$; ii. $n = 2$; iii. $n = 3$; iv. $n = 4$; v. $n = 5$; vi. $n = 6$.

9. (15 points) Conjecture a formula for $L_{n-1} + L_{n+1}$. Use induction to prove the correctness of your formula.

10. Let $D = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31\}$ and $R = \{2, 4, 6, 8, 10\}$.

(5 points) How many functions $f : D \to R$ exist?

(5 points) A **constant** function is one with $f(x) = c$ for all $x \in D$ where c is a constant. How many constant functions $f : D \to R$ exist?

11. Let $D = \{1, 2, 3, 4, 5, 6, 7\}$ and $R = \{a, b, c, d, e, f, g, h, i, j, k\}$.

(5 points) How many functions $f : D \to R$ exist such that odd numbers are mapped to consonants and even numbers are mapped to vowels?

(5 points) How many one-to-one functions $f : D \to R$ exist such that $f(3) = k$ and $f(5) = a$?