Infinite Sets

Any two sets that can be placed in a one-to-one correspondence must have the same number of elements and must be equivalent sets.

An **infinite set** is a set that can be placed in a one-to-one correspondence with a proper subset of itself.

Showing a one-to-one correspondence of infinite sets requires showing the pairing of the general terms in the two infinite sets. Consider the set $\{6, 11, 16, 21, 26, 31, ...\}$. The general term in this set is $5n + 1$.

A set is **countable** if it is finite or if it can be put in one-to-one correspondence with the counting numbers. All infinite sets that can be put in one-to-one correspondence with the counting numbers have cardinal number **aleph-null**, denoted $\aleph_0$.

We can show that the set of even numbers and the set of odd numbers can each be placed in one-to-one correspondence with the counting numbers. From this we reason that

$$\aleph_0 + \aleph_0 = \aleph_0$$

Consider a hotel with infinitely many rooms. If all the rooms are full we can reassign rooms so that the guest in room n is moved to room $2n$. Then we will have more rooms available - all the odd numbered rooms are available!

Cantor showed that there are different orders of infinity. Sets that are countable and have cardinal number $\aleph_0$ are the lowest order of infinity. Cantor showed that the set of integers and the set of rational numbers are infinite sets with cardinality $\aleph_0$. Cantor also showed that the set of real numbers could not be placed in one-to-one correspondence with the counting numbers and that they have a higher order of infinity, called the **cardinality of the continuum.**

Show that the set $\{12, 13, 14, 15, 16, ...\}$ is infinite by placing it in a one-to-one correspondence with a proper subset of itself.

The general term in this set is $11 + n$. $\{13, 14, 15, 16, ..., 1 + (11 + n), ...\}$ is a proper subset. We map $11 + n \rightarrow 1 + (11 + n)$.

Show that the set $\{4, 8, 12, 16, 20, ...\}$ is infinite by placing it in a one-to-one correspondence with a proper subset of itself.

The general term in this set is $4n$. $\{8, 12, 16, 20, ..., 4n+4, ...\}$ is a proper subset. We map $4n \rightarrow 4n+4$.

Show that the set $\{50, 51, 52, 53, 54, ...\}$ has cardinality $\aleph_0$ by establishing a one-to-one correspondence with the counting numbers.

The general term is $n+49$. We map the counting number n to $n+49$.

Show that the set $\{3, 6, 9, 12, 15, ...\}$ has cardinality $\aleph_0$ by establishing a one-to-one correspondence with the counting numbers.

Show that the set $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, ...\}$ has cardinality $\aleph_0$ by establishing a one-to-one correspondence with the counting numbers.

Show that the set $\{0, -1, 2, -4, 8, -16, 32, -64, ...\}$ has cardinality $\aleph_0$ by establishing a one-to-one correspondence with the counting numbers.

Show that the set $\{(1, 1), (4, 8), (9, 27), (16, 64), ...\}$ has cardinality $\aleph_0$ by establishing a one-to-one correspondence with the counting numbers.