A Brief Overview of Set Theory

I. Vocabulary

A ***set*** is a collection of objects.

Each object in a set is called an ***element*** or ***member***.

A set is ***well defined*** if its contents can be clearly determined.

There are 3 methods commonly used to indicate a set:

1) ***Description***
2) ***Roster forms*** - the elements are listed inside braces {}
3) ***Set builder notation***

The ***natural numbers*** or ***counting numbers*** are denoted by $\mathbb{N} = \{1, 2, 3, 4, 5, ...\}$.

The word ***inclusive*** means to include the end points.

The word ***exclusive*** means to exclude the end points.

The symbol $\in$ means is an element of. Similarly, $\notin$ means is not an element of.

A set is ***finite*** if it either contains no elements or the number of elements in the set is a natural number.

A set that is not finite is ***infinite***.

A set A is said to be ***equal*** to a set B if and only if set A and set B contain exactly the same elements.

A ***cardinal number*** of set A, symbolized by $|A|$ or $n(A)$, is the number of elements in set A.

A set A and a set B can be placed in ***one-to-one correspondence*** if every element of set A can be matched with exactly one element of set B and every element of set B can be matched with exactly one element of set A.

The set that contains no elements is called the ***empty set*** or ***null set*** and is denoted by $\emptyset$ or by {}.

A ***universal set*** is the set that contains all the elements for any specific discussion. It is denoted by U.

An ***ordinal number*** describes the relative position that an element occupies.

II. Examples

Express in roster form the set of states in the United States whose names begin with the letter I.

{Iowa, Illinois, Indiana, Idaho}

Determine whether the set of the nicest entertainers is well defined.

Is the set of multiples of 6 between 0 and 90 finite or infinite?

Is the set of even numbers greater than 19 finite or infinite?

John Grisham has written 12 books. Is the number cardinal or ordinal?

Lincoln was the sixteenth president of the US. Is the number cardinal or ordinal?

Express $D = \{5, 10, 15, 20, ...\}$ in set builder notation.

$D = \{5n \mid n \in N\}$

Write a description of the set {Bashful, Doc, Dopey, Grumpy, Happy, Sleepy, Sneezy}.

The names of the seven dwarfs in *Snow White*.

Write a description of the set $E = \{x \mid x \in \mathbb{N} \text{ and } 5 < x \leq 12\}$.

The natural numbers greater than 5 and less than 13.

$A = \{2, , 4, 6, 8\}$ What is $|A|$?

Let $A = \{x \mid x \in \mathbb{N} \text{ and } x > 2\}$ and $B = \{x \mid x > 2\}$.

A) Write a description of set A and set B.
Set A is the set of natural numbers greater than 2. Set B is the set of all numbers greater than 2.

B) Explain the difference between the two sets.
Set A is restricted to the natural numbers and set B is not.

C) Write set A in roster form.
$A = \{3, 4, 5, 6, 7, ...\}$

D) Can set B be written in roster form? Explain.
If we assume that the universe is all real numbers than the elements cannot be listed.

Subsets

I. Vocabulary and Concepts

Set A is a ***subset*** of set B, denoted $A \subseteq B$, if and only if all the elements of set A are also elements of set B.

We use $\not\subseteq$ to indicate that A is not a subset of set B. To show that set A is not a subset of set B we must find an element in A that is not in set B.

Set A is a ***proper subset*** of set B, denoted $A \subset B$, if and only if all the elements of set A are elements of set B and $A \neq B$. Set B must contain at least one element not in set A.

The empty set is a subset of every set including the empty set.

The number of distinct subsets of a finite set A is 2^n where n is the number of elements in A.

II. Examples

Let $A = \{\text{Pepsi, Mountain Dew, Coke, Sprite}\}$. Let $B = \{\text{Pepsi, Coke}\}$. Determine whether $A = B, A \subseteq B, A \subset B, B \subseteq A, B \subset A$.

List all subsets of the set $C = \{a, b, c\}$.

How many subsets should we list?
How many proper subsets should we List?

Jason Jackson is considering having his computer upgrades. He can leave the computer as it is, or he can upgrade any of the following sets of items: RAM, modem, video card, hard drive, processor, sound card. How many possible options for upgrading does Jason have?

A ={RAM, modem, video card, hard drive, processor, sound card}
$n(A) = 6$
The number of subsets of A is $2^6 = 64$.