Homework

Section 2.1: 1, 2, 4, 5, 10-17, 19a, 19b, 23, 28, 29, 30
1. Prove $\sqrt{5}$ is irrational. Hint! Follow the proof from class that $\sqrt{2}$ is irrational.
Section 2.2: 1, 2, 3, 4, 12, 13, 15a, 16, 17a, 19, 20, 25-27, 32, 33, 34, 35, 36, 38, 39, 41
Section 2.3: 1, 2, 4, 5, 8-13, 16, 17
Section 2.4: 1, 2, 3, 5a, 5d, 5f, 6a, 6b, 6c, 6d, 6e, 9a, 9b, 9c, 9d, 9e, 10a, 10b, 13, 14, 16, 23, 27, 31, 32, 40, 42

Section 4.1: 3, 4, 5, 9, 14, 18, 20, 21, 31-34
1. Use induction to prove $\sum_{i=1}^{n} 2i - 1 = n^2$ for all $n \in Z^+$.
2. Use induction to prove $(2n)! > 3^{2n+1}$ for $n \geq 4$.
3. Use induction to show that $6|(n^3 - n + 12)$ for all $n \in Z^+$.
4. Use induction to prove $\frac{2^{3n} + 3^{n+2}}{5}$ is an integer for all positive integers n.
5. Use induction to prove $\frac{2^{3n} - 22}{7}$ is an integer for all positive integers n.

Section 4.2: 3, 5, 6, 7, 10, 12, 32
Section 4.3: 1, 2, 12, 13, 24a, 24b

1. Again, a group of friends, which includes numerous engineers and non-engineers, are gathering for a social occasion. At one point, some collection of n of the partygoers will sit in n consecutive chairs. In this case, we always want two engineers sitting next to one another so they will have someone to talk shop to and we always want two non-engineers sitting next to one another. Also, each row will always begin with a non-engineer but may end with either type of individual. Let S_n be the number of ways of seating n people in these n chairs? Construct all possible arrangements and compute S_n for all values up to $n = 5$. Find and prove the correctness of a formula for S_n.

2. This time, there are n chairs and some collection of people (including none) will sit in the seats but there will always be at least one empty chair between any two people. Let A_n be the number of antisocial ways to seat some number of people in these n seats as described. Construct all possible arrangements and compute A_n for all values up to $n = 4$. Find and prove the correctness of a formula for A_n.

Section 5.1: 1-4, 6-13, 19, 23-25, 30, 32, 33, 46, 47
Section 5.2: 1-5, 9, 17, 18, 33
Section 5.3: 1-8, 10, 13-17, 27, 28, 30, 31
Section 5.4: 1b, 2b, 3-9, 15 (**Hint!** Use Binomial Theorem to prove $\sum_{i=0}^{n} \binom{n}{i} = 2^n$), 21, 22, 24, 28, 29

1. Use the Binomial Theorem to prove $\sum_{i=0}^{n} (-1)^i \binom{n}{i} = 0$.

2. Use a combinatorial proof to show $\binom{3n}{2} = 3\binom{n}{2} + 3n^2$ for $n \in Z^+$.

3. Use a combinatorial proof to show $\binom{2n}{n} = \sum_{i=0}^{n} \binom{n}{i}^2$ for $n \in Z^+$.

Section 5.5: 1-3, 5-12, 17, 19, 30-32, 38
Section 6.1: 1, 3, 6-14, 23
Poker Handout: 1-12
Section 7.5: 1-6, 10-12, 15

1. In a survey of 100 people, 73 drink Coke or Pepsi. Of those surveyed, 33 exclusively drink Coke while 17 exclusively drink Pepsi. How many people drink both Coke & Pepsi? How many people drink neither Coke nor Pepsi?

2. Of 200 movie patrons surveyed, 78 always bought candy or popcorn. Of those people, 45 always bought candy while 51 always bought popcorn. How many people always buy both popcorn and candy?

3. At a local high school, 50 students are on the football team, 19 on the basketball team, and 25 on the baseball team. There are 12 students who play both football and basketball, 18 who play both football and baseball and 7 who play both basketball and baseball. There are 4 students who play all three sports. How many students play on at least one of football, basketball or baseball?

4. Can the following scenario occur? Explain. There are 95 students who play at least one of football, basket ball and baseball. There are 64 football players, 28 basketball players and 29 baseball players. There are 17 students who play both football and basketball, 13 students who play both football and baseball and 12 students who play both basketball and baseball.

Section 9.1: 3-9, 13, 16
Section 9.2: 1-5, 7-10, 18, 20 (a,b,c,d,e), 21-25, 26 (a-c), 29 (a-d), 30, 31 (a-d), 32-35, 37, 47 (a-c), 48, 49, 53 (a,b), 54-56