1 Computations

1. $10! =$ 2. $\frac{500!}{502!} =$ 3. $\binom{18}{4} =$ 4. $\frac{\binom{2n}{2}}{\binom{2n}{3}} =$

5. Without calculating a final answer, show $16! = 14!5!2!$.

6. Let $A = \{1, 2, 4, 6, 8, 10\}$ and $B = \{1, 3, 6, 9\}$ with the universal set $S = \{1, 2, ..., 10\}$. Construct
i. $A \cup B$;
ii. $A \cap B$;

iii. $A - B$;

iv. $B - A$; Give a concise mathematical reason why this is not redundant to part iii.
v. $A \oplus B$;

vi. $\overline{A \oplus B}$;

vii. $\overline{A} \oplus \overline{B}$;

viii. $\overline{A \cup B}$;

ix. $\overline{A} \cap \overline{B}$. Give a concise mathematical reason why this is redundant to part viii.

7. $\sum_{i=3}^{7} 2i - 4 =$

8. $\sum_{i=1}^{n} 2^i \binom{n}{i} =$

2 Problems

1. Six men and seven women stand in a line at the bookstore.
   (a) How many arrangements of these people are possible?

   (b) How many arrangements of these people are possible if the men stand in succession?

   (c) How many arrangements of these people are possible if two women stand at the very front of the line and two men are at the very end of the line?

   (d) How many arrangements of these people are possible if the men and women must alternate positions within the line?

2. A local fast-food outlet offered a variety of meal combinations. Every meal combination included a sandwich, an order of French fries, and a soft drink. Suppose there are 6 different sandwiches, 3 different sizes for French fries orders, and 8 different soft drinks to choose from. How many meal combo orders must be placed to assure that at least one meal combo is ordered twice? Explain.

3. Five brothers each have the same set of seven hats, distinguished only by color. Each has a white hat, a black hat, an orange hat, a green hat, a yellow hat, a maroon hat, and a red hat.

(a) How many ways are there for each of the brothers to all choose hats of different colors?

(b) At a recent family reunion, four of the brothers were seen wearing the same color hat while the fifth brother wore a hat of a different color. In how many ways could this have occurred?

4. How many unique arrangements are there for the letters in the word *magic*?

How many unique arrangements are there for the letters in the word *illusion*?

How many unique arrangements are there for the letters in the word *abracadabra*?

5. At Bunion College, a small undergraduate institution in the Midwest, every student must select a password to use to enter the college computer network. In creating a password, the following restrictions must all be met.

The password must contain only digits or lowercase letters of the alphabet, but not both.

The password must be four or five characters in length.

The password must begin with b, q, x, 3 or 7.

How many different passwords are possible under these restrictions?

6. The Quik-Stop convenience store has jars of 15 different types of candy available for purchase in any desired quantity. Veronica intends to select 6 pieces of candy to purchase.

i. How many ways can Veronica make her selection if each piece of candy is different?

ii. How many ways can Veronica make her selection if she will select four pieces of one type of candy and two pieces of another type of candy?

iii. How many ways can Veronica make her selection if she will select three pieces each of two types of candy?

7. A bag of plain M&M's may contain blue, brown, orange, red and/or yellow candies. How many different color combination of candies are possible in a bag of 23 M&M's?

8. Susan buys an economy pack of fifty pens. The pens are identical except for color. There are ten of each of five different colors.

How many different ways can Susan select four pens of different colors to take to work?

How many different ways can Susan select ten pens to take to work?

How many different ways can Susan select ten pens to take to work with at least one of each color?

How many different ways can Susan select twelve pens from her economy pack to take to work?

9. Provide an algebraic proof that $\binom{3n}{2} = 3\binom{n}{2} + 3n^2$.

10. Provide a combinatorial proof to show that $\binom{3n}{2} = 3\binom{n}{2} + 3n^2$.

11. Let $D = \{2, 3, 5, 7, 11, 13, 17, 23, 29, 31\}$ and $R = \{2, 4, 6, 8, 10\}$.
i. How many functions $f : D \to R$ exist?
ii. How many functions $f : R \to D$ exist?
iii. How many functions $f : D \to D$ exist?
iv. How many constant functions $f : D \to R$ exist?
v. How many constant functions $f : R \to D$ exist?
vi. How many functions $f : D \to R$ exist such that $f(5) = f(11) = 8$?

12. Consider playing Blackjack with a single deck of cards. When selecting two cards at random what is the probability of blackjack (Ace and one of a 10, Jack, Queen or King)?

13. Consider playing Blackjack with a single deck of cards and two distinct jokers. When selecting two cards at random what is the probability of blackjack?

14. In poker, what is the probability that you are dealt
i. a flush;

ii. a full house?

15. Use induction to prove $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

16. Prove $\overline{A \cup B} = \overline{A} \cap \overline{B}$.