Math 1106 – Elementary Applied Calculus
Bittinger and Ellenbogen's *Calculus and Its Applications, 9th Edition*
Post-Lecture Notes for Chapter 1, "Differentiation"

After each lecture over the material in Chapter 1, some summary and supplemental thoughts about the topics discussed in the lecture will be added to this document. However, not everything said in the lecture will be listed here, and students are expected to attend each lecture to obtain all pertinent information.

Added after Day 2 lecture, August 19, 2009
(Section 1.1, Limits: A Numerical and Graphical Approach)

Today's lecture introduced the strange idea of "limit of a function." For a specific x-value (like the book, I'll generically call that specific value "a" instead of a particular number), the limit's a number, and this number's notation is $\lim_{x \to a} f(x)$. Our challenge is, first of all, to determine if there is a limit; and if there is, to nail down its numeric value. Remember, because it's "like" an f(x), it is obviously saying something about the height of the graph for that particular value of x = a. But the sometimes-confusing thing for students is that there are examples where the limit is exactly equal to f(a), and others where it isn't, and still others where that limit does not exist!

The important thing to remember is that this limit number, while it speaks to the height of the function at x = a, it isn't saying what that height is. No, it's saying what the height ought to be. "Huh? Ought to be?" What does that mean? It means, what are the really close neighboring points of the graph saying about their own heights? And, regardless of what the numeric value of f(a) really is, is there a height where a single dot would close the gap between the neighbors on either side? Or is the gap so huge (i.e., the left-limit and the right-limit exist, but they don't agree) that a single dot couldn't bridge the gap?

I know, this "limit of a function" is a strange new thing that can be both confusing and mysterious. One moment you think it makes sense, and then the next you're puzzled again. But hang in there, and in 2 or 3 more sessions, you'll see exactly where we (and Isaac Newton) will be making important use of this strange new beast to achieve our goal of being able to define the slope of a function that is something other than a straight-line ("linear") function.

If you're not yet set up in MathXL, then get crackin'! Doing your homework in a timely manner is so very important to success in this subject. And I don't just want you to "pass" this course; I want you to earn an "A" and I know you can do it if you've got a proper plan for success. Don't procrastinate!

Added after Day 3 lecture, August 24, 2009
(Section 1.2, Algebraic Limits and Continuity)

We need to remember that in algebra a topic that was discussed was an arithmetic for functions; new functions can be built from other functions through the use of addition, subtraction, multiplication, and division. (Even raising a function to a power — either a positive integer, or a fractional, power — yields a new function. Please remember that a fractional power is equivalent to taking a "root" of that function.) Well, for such manufactured functions their limit for a particular value of x is obtained from the constituent functions' limits at that value of x in the same arithmetic manner. The only "interesting" case is for the so-called "rational" functions that arise from dividing one function by another, and that's when the limit of the denominator function is equal to zero.

Most of the functions we study are continuous everywhere, meaning that they can be graphed without lifting your pencil off the paper. Anywhere a function is continuous, the limit there for that value of "x" ("what the neighbors of "x" are saying about their y-values", or "where the y-values for the x-values near "x" seem to be headed) is always equal to whatever the y-value associated with the given "x". So you'd just plug "x" into the equation to get the limit. Very easy.

The only functions we've run into in this course (so far) where that isn't true (that is, functions where there are values of "x" having discontinuities – breaks) are:

- the manufactured piece-wise functions (their "pieces" are continuous, but where they "stitch together" are places where there is either a "hole" or a "giant gap")
- the "rational" functions (numerator and denominator are both continuous functions), but these only have discontinuities where plugging the "x" into the denominator part gives a result, for that part, of zero.

The key to evaluating the limit of a rational function when the value of x you're substituting makes the denominator equal to zero is to look for a way to factor the numerator and/or denominator so as to eliminate the troublesome common factor that is causing the zero. Once you've done that, then re-evaluate the limit at the x-value of interest, and if that gives you a definite number, that's the limit. Voila!

A classic example is given by the authors as this, where you're to find $\lim_{x \to -3} \frac{x^2 - 9}{x + 3}$.

It's simple: first determine the limits of the two constituent functions, also when x is approaching -3. Those two are continuous functions, so getting the limit is easy, just by evaluating both functions when x is equal to -3. The problem is that the bottom function turns out to equal zero when x = -3, so getting the limit of the rational function seems to turn messy. But keep the faith! The trick is to first check out the limit of the top function — it, too, turns out to be zero. And there's the great thing: whenever this is the case, when the fraction you get turns out to be the so-called "indeterminate form" of $\frac{0}{0}$ then you can "take it to the bank" that there is a limit — and there's that algebraic process I spoke about in the previous step to handle this. (If the top function's limit is something other than zero when the bottom function's limit is zero, then that's an easy answer: there would be no limit.)

So, we go about finding that limit in this manner, algebraically:

$\lim_{x\to -3}\frac{x^2-9}{x+3}=\lim_{x\to -3}\frac{(x+3)(x-3)}{(x+3)}=\lim_{x\to -3}(x-3)=-3-3=-6.$ (In the last step, since x-3 is a nice continuous linear function, its limit at x = -3 is just whatever it's equal to when x = -3.)

So remember this about continuous functions, or a function that "is continuous at a specific value of x": its y-value there must equal its limit there.

Hope you're doing okay with your MathXL homework.

For years I've driven my family to Virginia for holidays and vacations. The trip is 550 miles and I've got it down to a science: I drive it in 11 hours, which means I make it with an average speed of 50 miles per hour. The graph of my actual progress (miles versus hours) would be only a little bit different from the graph on page 113, the one that shows the productivity of the garment factory owned by Raggs, Ltd. It always goes upward, with some places it flattening out (when we take breaks and stop for meals) and some places it climbs quite steeply (as I put the car on cruise control and zoom on down the interstate).

If you add a straight line from the beginning point of the Raggs, Ltd., graph (8 a.m.) to the ending point (12 p.m.), that straight line's slope value is a measure of overall productivity of the garment factory for the entire morning shift: the average number of suits made per hour. (For my trip, the measure of productivity is called "average miles per hour, or average speed.")

Productivity (average rate of change) at the garment factory can even be measured for various segments of the morning. For instance, at the top of page 123, it's calculated that the productivity between 9 a.m. and 11 a.m. is equal to 22 suits per hour. Remember, though, that's an average …. there's no guarantee that during the one-hour span between 9:30 a.m. and 10:30 a.m. that the workers actually produced 22 suits. (They may have taken a coffee break; that's just like my 50 m.p.h., which includes our lunch stop at Wendy's that day.)

The slope of any of those straight blue lines that connect pairs of points on the graph is a measure of productivity during each one-hour span of time. Turning to functions in general, the big shaded box on page 123 lays out the definition of the process of calculating the slope of such a straight line. And to no one's surprise (I fervently hope!), the calculation of the slope is remembered to be simply "rise" divided by "run" … "rise over run."

The numerator and denominator are simply the amount of change in each quantity involved. For the garment factory, the numerator was the change in the total number of suits that had been produced during the relevant time period, while the denominator was the matching change in the number of hours from the beginning of the period until the end of the period. Always subtract the beginning amount from the ending amount both top and bottom, and you'll be assured of getting the right answer. (If you don't obey that rule, your answer may turn out to be wrong by a factor of -1 [that's minus one].)

The graph on page 126 works with the general case of a function, and calculating the slope of the straight line that connects two points on the graph (red line) of the function. There's some practice (Example 3) in calculating the slope for a particular function, between various pairs of points.

But then, at the bottom of page 127, the general graph of a function back on page 126 is redrawn. It looks so very much like the graph on page 126, but there are some important differences. And here it's important to note, that the coordinates of the two points on the red line have been stated in a new way. Instead of the first one being (x_1, y_1) and the second one (x_2, y_2) [or, if you're comfortable with functional notation, those could be written equivalently as $(x_1, f(x_1))$ and $(x_2, f(x_2))$], as on page 126 …. now on page 127 the first x-value is simply called "x" and the second point's x-value is called $x+h$ [that's "x plus h"]. In other words, if the amount it takes to get from x_1 to x_2 is h [that is, $x_2 - x_1 = h$], then we know that $x_2 = x_1 + h$ …. and since we're renaming x_1 to simply x, that means x_2 gets the new name $x+h$.

Well, if the x-coordinate of the two points on the graph are called x and $x+h$, then their y-coordinate are respectively $f(x)$ and $f(x+h)$, right?

Okay, using these new names for the coordinates of the two points, let's calculate the slope (symbolically) of the straight line between the two points on the graph on page 127. It's the rise over the run:

$$\frac{\text{rise}}{\text{run}} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}.$$

Make special notice of the denominator, how it really simplifies because x gets subtracted from $x+h$. It turns out to be simply "h," of course! That's great! It will simplify matters enormously to use this new way of describing the coordinates of the second point on the graph. For one thing, the denominator of this expression (it's called a quotient, remember?) is simply equal to "h".

This quotient, that represents the slope of the straight line between two points on the graph of the function, is called "the difference quotient." Memorize this expression, both its name and its representative fraction!!! Work on fully understanding what it represents, and what the two y-values in the numerator represent. Don't go getting confused by $f(x+h)$. You MUST understand that, and make your peace with it. It's the y-value that the function named f associates with the x-value of the second point, which is the value of x you get by adding the amount "h" to the x-value of the first point. And, because the second point is distinct from the first point, and because it's a true function we're working with (that means it must "pass" the Vertical Line Test), then there is absolutely no way that h can be equal to zero.

Oh, and any straight line that connects two points on a graph has a special name: it's a secant line. (Secant is the Latin word for "to cut". The line "cuts" the graph.)

The rest of Section 1.3 involves substituting the actual defining expression for various specific functions into the generic difference quotient, and "simplifying" what you get from that. Sometimes it'll be dirt simple; sometimes you'll need to recall some of the "tricks" we learned in basic algebra (for instance, Example 7 which starts at the bottom of page 129 uses one of those tricks). Although the author doesn't do a good job of clearly defining what's meant by "simplification," you've got my word for it: It means to "get the 'h' out!" Meaning, once you've used the definition of the function *f* to work out the difference quotient, the goal is to factor the numerator in such a way that the "h" in the denominator will factor against another copy of it in the numerator and get cancelled out.

Remember the look of the difference quotient: $\dfrac{f(x+h)-f(x)}{h}$. We'll be encountering it frequently during the semester, and you'll want to treat it like an old friend whenever you see it.

Added after Day 5 lecture, August 31, 2009

(Section 1.4, Differentiation Using Limits of Difference Quotients)

This Section is where the two major concepts we've seen up to now in this course are married up: the limit of a function, and the difference quotient for a function. The result is called the derivative function. If the original function is known as "f", then the new function is:

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Notice how the new function takes a name that is almost identical to the original function, but with a tick-mark ("prime" or "single quote") attached to the name. This new function calculates the slope of the tangent line to the original function at the specific value of x. While the difference quotient by itself gives an average rate of change (of the y-values versus the x-values) between two points on the graph of the original function, the derivative gives the instantaneous rate of change at a single point. In situations where the basic function f(x) is measuring position (or distance traveled) versus the passage of time, then the slope of the tangent line is the instantaneous speed. Stay tuned! Newton (and Leibniz) have now opened the door to some really useful applied mathematics!

Added after Day 6 lecture, September 2, 2009

(Section 1.5, Differentiation Techniques: The Power and Sum-Difference Rules)

The first order of business today was to go over the various notations that are used for the notion of a derivative. I told about the historic reasons for each one, and the need for knowing the variants: textbook authors prefer one or the other, and you may encounter different notations as you study different courses during your college career. So you have to "speak the lingo." In the following notes, I'll bounce back and forth between the "prime" notation and the d-y-d-x notation popularized by the 17th century mathematician named Leibniz.

Now it can be revealed: There are several "rules of differentiation" that make it no longer necessary to evaluate the limit of the difference quotient to get the equation for a derivative function, f'(x), from a given function f(x). These rules are simple to apply, and it can become downright fun (he said with a straight face). As each rule is revealed, we expand the population of functions that we can differentiate without having to "go back to the well" — meaning, without having to drag out and struggle with the official definition of the derivative: the limit of the difference quotient for the function, taken to the limit, as h approaches 0.

Rule 1: The Power Rule. It handles all functions that are simply a fixed power of the input variable, for instance $y = x^7$. Such a function is called a "power function." And it doesn't have to be just positive integers — any fixed power will work. And the rule, for the record, is this: $\frac{d}{dx}x^k = kx^{k-1}$, where *k* stands for the fixed power being used. Isn't that simple? So, using the specific example about 5 lines back, $\frac{d}{dx}x^7 = 7x^{7-1} = 7x^6$.

Rule 2: The Constant Function Rule. This is amazingly simple; so simple, in fact, that you might wonder why we even need to announce it as a rule. Well, wait until you see the example I give after Rule 4, and you'll see why. So, what's a constant function? Something like $f(x) = -9$ is an example of a constant function. No matter what the input ("*x*") value is, the function assigns the value -9 to be its corresponding y-value. So, when it's graphed, it's just a horizontal line. What's the slope of a horizontal line? It's zero! So, the Constant Function Rule just says that $\frac{d}{dx}c = 0$, for any given constant function that draws the horizontal line at level "c".

Rule 3: The Constant Multiplier Rule. Look at a function like $g(x) = 7x^5$. It's not exactly a Power Function because of that coefficient multiplier of 7. But this rule will allow us to quickly calculate its derivative, since we know how to get the derivative of the power function represented by simply x^5. The rule says this: $\frac{d}{dx}c \cdot f(x) = c \cdot \frac{d}{dx} f(x)$. That's not so cryptic; the function above is definitely such a function. It consists of a constant (the "7") times another function ($f(x) = x^5$). So, $\frac{d}{dx}7x^5 = 7\frac{d}{dx}x^5 = 7 \cdot 5x^4 = 35x^4$ (after multiplying out the multiplier of 7 times the 5 that developed out of the derivative of the function named *f*).

Rule 4: The Sum-Difference Rule. Here's a rule that helps to expand the world of functions we'll have differentiation rules for. Remember how functions can be built by adding together (or subtracting one from another) two functions? Well, it turns out that the derivative of such a function is simply built the same way from the derivatives of the constituent functions. Thus, the rule is stated like this: $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$. So, depending whether the original function is built with addition, the derivative is built from addition; and if by subtraction, then by subtraction. That's the implied meaning of using the double-symbol of $\pm$. And notice how I sneaked in two different notations for derivative there: the one from Leibniz, and then the so-called "prime" notation for functional notation. An example of a more complicated function that's built up through sums and differences is this: $\frac{d}{dx}[4x^{-6} - 3.5x^2 + 5x - 34.9] = -24x^{-7} - 7x + 5$. The first two terms of the derivative probably gave you no problem (other than having to remember that one less then negative-six is negative-seven). The third term, 5x, should be thought of as $5x^1$, and then its derivative is $5x^0$, but $x^0 = 1$, so the derivative of 5x is 5. And the fourth term, -34.9, is just a constant — and Rule 2 says that's equal to zero, so there's no sense even writing down anything for its derivative, since adding (or subtracting) zero doesn't change anything.

Stay tuned: I promised that the practice test for the First Test of the semester will appear on my website on Labor Day. There'll be two versions: one just the practice test, and the other a fully-annotated solution for that practice test. Get to work on your homework! It's going to be due at midnight the night before the first test.

Added after Day 7 lecture, September 9, 2009

(Section 1.6, Differentiation Techniques: The Product and Quotient Rules)

The first 4 rules covered in Section 1.5 apply to many simple polynomial functions, and can even be used for functions that are products or quotients of polynomials, provided the underlying polynomials are not too complicated. For instance, suppose your task is to find the derivative for a function such as $f(x)=(2x+7)(3x+6)$. At that point, the best you could do would be to multiply ("foil it out") to get $f(x)=6x^2+33x+42$ and then it's easy to apply the Power Rule, Sum-Difference Rule, Constant Multiplier Rule, and the Constant Function Rule (and you get so good at doing all those that you don't even think of them as separate rules) to determine the derivative: $f'(x)=12x+33$.

But these two new rules for differentiation handle the more complicated product and quotient functions with great ease. You must be able to remember and use these rules to derive the rate-of-change ("derivative") function for a given function.

The **Product Rule** applies when a function (like the example above) can be written as the product of two other, simpler functions: If $f(x)=g(x)\cdot h(x)$ then $f'(x)=g(x)\cdot h'(x)+g'(x)\cdot h(x)$. Study that carefully, and get familiar with its "mix and match" quality. It's a pair of products added together. Each pair involves one of the original factors times the other factor's derivative. Order here does not so much matter, since both multiplication and addition are commutative operations ("commutative" just means, for example, that 5 times 8 is the same as 8 times 5, and 3 plus 7 equals 7 plus 3).

The Product Rule gives us an alternate way to develop the derivative in that previous example, where $f(x)=(2x+7)(3x+6)$. It would be: $f'(x)=(2x+7)(3)+(2)(3x+6)$. See how the 3 is the derivative of the second function, while 2 is the derivative of the first function?

When you multiply out that result, it comes out to be exactly equal to what was obtained before: $f'(x)=12x+33$. In the suggested homework exercises from the book (which match a lot of the homework you'll encounter on MyMathLab) that I've listed above, you'll get lots of practice with the Product Rule. Some of them even tell you to get the derivative "in two ways," just so you'll see that the answer comes out the same either way you do it.

For functions that are ratios of other functions (another word for ratios is to call them quotients), the **Quotient Rule** is a bit trickier to remember. The author of the textbook lays out a systematic way to go about describing the steps. Here's my way.

When I must get the derivative of a function that looks like this, $Q(x)=\dfrac{N(x)}{D(x)}$ (notice the unsubtle way that the individual functions are named: the **Q-**uotient function is the ratio of the **N-**umerator function and the **D-**enominator function!), then my first steps begin with using the Denominator function in the answer, so that it starts off looking like this:

$$Q'(x) = \frac{D(x)\ldots\ldots\ldots\ldots\ldots\ldots}{[D(x)]^2}\ .$$

Notice that in the derivative, its denominator is the original denominator **squared**.

Then, complete the numerator portion of the answer as if it's a mix-and-match in the style of the Product Rule, **except** that you'll use subtraction instead of addition. And, it's very important in my way of doing it that you start off the derivative's numerator with the **D-**enominator function for this to work out correctly!

$$Q'(x) = \frac{D(x)N'(x) - D'(x)N(x)}{[D(x)]^2}$$

Yeah, multiplication is still commutative, so it doesn't matter what order each factor appears in a product. But it is highly important which product appears first, since subtraction is not commutative! Screw it up, and you'll have the wrong answer!

See you Monday, for the review prior to Wednesday's test. Please study carefully the practice test I've made available on my website, and come to class with any questions you develop from studying it.

Added after Day 8 class meeting, September 14, 2009
(review for the test on Wednesday of this week)

The class was devoted entirely to answering questions about the material in Chapter 1, up through Section 1.6. Questions about the homework that's due on Tuesday night (by midnight) were also covered. It was a good session. If you weren't there, where were you?

You can email me, or visit me in my office (Willingham Hall Room 114), or get help at the Math Lab on the 4th floor of the library. Don't waste this chance to get a great score on the first test of the semester!

Added after Day 9 class meeting, September 16, 2009

Today was devoted to taking Test #1.

Added after the Day 10 class meeting that never was, September 21, 2009

What a day! Before I could even meet the first class of the day (at 12:30 pm), the rains came, and the classroom in Willingham Hall began flooding! And then the University cancelled classes for the day. Several of the 12:30 class students showed up, and I gave them back their tests. Then I waited out the crush of cars trying to get out of the parking garage before venturing out across campus under my umbrella, and found that my car was safe but its rear wheels resting in a very big puddle of water. Today would have been devoted to going over the test results, as well as learning about the Chain Rule (Section 1.7 of the book).

Added after the Day 11 class meeting that never was, September 23, 2009 (the following notes will make more sense after I lecture about the Chain Rule and Higher Derivatives ... on Monday, September 28??? I hope!!)

I had great hopes that classes would meet today, but it's another day with the University closed, just like yesterday (Tuesday). I just knew I'd be able to make up the lost time and cover both the Chain Rule and Higher-Order Derivatives (Section 1.8) as well as going over the test results. Well, hopefully school will be open next week! For those who are diligent about preparing for class, here are notes about the Chain Rule — please come to class with some familiarity with the topic, and your questions.

The Chain Rule is the most important of the rules of differentiation. It gives a way to derive the slope function from functions that the other rules don't completely address. But to use it, you must be able to discern when a given function is composed of (or "built up from") a chain of other functions. This is a topic you were exposed to in algebra, and now you get to see why! I myself wish they'd call it "chaining functions" since the rule for differentiation is called "The Chain Rule."

For example: the function $f(x) = (x^2 - 3x + 7)^{14}$ is composed of the chaining together of two functions. Function *f(x)* is composed of (is the "chaining together" of) the two functions $g(x) = x^{14}$ and $h(x) = x^2 - 3x + 7$. This chaining together is represented by writing $f(x) = g \circ h(x)$. This means that the input variable (represented by *x*) is first acted upon by the function named *h* to produce the intermediate result *h*(*x*); then this result is used as input to the function *g*, which raises the intermediate result to the 14th power. In other words, $f(x) = g \circ h(x) = g(h(x)) = (h(x))^{14} = (x^2 - 3x + 7)^{14}$. Think of the symbol $"\circ"$ as a single link in a chain, indicating the linking of the two functions *g* and *h*. Be sure that you notice that the order in which the linking is done is critically important. In most cases, and indeed in this case, the function obtained through the linking $g \circ h$ is not the same as the one obtained by linking $h \circ g$. Try it, and see what the opposite chaining of *h* and *g* produces! (In other words, function chaining, a.k.a. "function composition", is not commutative.).

Once you're adept at decomposing a function into its constituent parts (the building-block functions that link together to obtain the given function), then the chain rule gives a very quick way to get the derivative, provided you can differentiate the two linking functions.

The chain rule says when $f(x) = g \circ h(x) = g(h(x))$, then $f'(x) = g'(h(x)) \cdot h'(x)$. Another way to think of this chaining is to regard the function *h* as being the "inside" function and function *g* as the "outside" function. So, the derivative turns out to be the outside function's derivative (chained together with the original inside function) multiplied by the derivative of the inside function.

In this way, we "peel the onion" differentiating as we go: get the derivative of the outer function (and keep the inside function "inside"), peel off the outer function, and then multiply what you've got so far by the derivative of the remaining inside function.

So, let's get the derivative of the function given above ($f(x) = (x^2 - 3x + 7)^{14}$), where $g(x) = x^{14}$ is the outside function and $h(x) = x^2 - 3x + 7$ is the inside function. First, get the derivatives of the two functions *g* and *h*: $g'(x) = 14x^{13}$ and $h' = 2x - 3$. Then $f'(x) = g'(h(x)) \cdot h'(x)$ which is equal to $g'(x^2 - 3x + 7) \cdot (2x - 3)$ and then (by the definition of $g'(x) = 14x^{13}$) this becomes $14(x^2 - 3x + 7)^{13}(2x - 3)$.

Now, this chain rule's powerful and orderly application can be exploited with more complicated functions, functions that are built up by chaining together 3 or more functions that are its "links." Here's where the analogy to peeling an onion works, as we see the various layers getting differentiated and peeled off. Unfortunately, concrete examples of this are hard to come by yet, and will have to wait until Chapter 4. But working symbolically, suppose we have a function that is the chaining together of 4 functions, like this:

$$f(x) = g \circ h \circ m \circ r(x) = g(h(m(r(x)))).$$

Then the derivative of the function *f* is gotten by successively applying the chain rule, and the "peeling of the onion" can be seen here:

$$f'(x) = g'(h(m(r(x)))) \cdot h'(m(r(x))) \cdot m'(r(x)) \cdot r'(x).$$

Are your eyes watering yet?

The final thing that I will cover is the alternative version of the Chain Rule that the author talks about *very* briefly at the bottom of page 173. In fact, all he says is this:

The Chain Rule often appears in another form. Suppose that $y = f(u)$ and $u = g(x)$. Then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

Then there are a series of exercises (45 thru 50) on page 175 that pertain to this cryptic other form of the Chain Rule, and a couple of the exercises on your MathXL homework, too.

It's not hard to follow the bouncing ball. In each exercise, they give you two functions. The first one has as its input variable the letter "u" and the second one has "u" as its output variable with an input variable named "x". Get the picture? These are the two function machines – the one on the right is the "first" or "inner" function, while the one on the left is the "second" or "outer" function.

The instructions say for you to "Find $\frac{dy}{du}, \frac{du}{dx}$, and $\frac{dy}{dx}$." So all they're asking you to do is to first find dy/du – which is just the derivative of the function on the left (the outer function) – then find du/dx, which is the derivative of the function on the right (the inner function). Then multiply the two of them together. That's exactly how the Chain Rule works: the derivative of the outer function (with respect to the inner function) times the derivative of the inner function.

The really neat thing about this is that the third derivative it tells you to get is dy/dx – the derivative of the chained function with respect to the original input variable "x". And written as the author does at the bottom of page 173, it looks for all the world like those three symbols are fractions (which they really aren't). But check that line out again! If you treat them like fractions, the "du" in the denominator of one seems to cancel against the "du" in the other's numerator (just like fractions would) and what's left is dy/dx – which represents the derivative of the chained function!!!!!

Concerning Higher-Order Derivatives: Since the derivative of a function is itself a derivative, you can go to the next step and differentiate it. Isn't this fun!!!

Pay attention to the notation for the second derivative. Since we write the derivative in several different ways, make sure you can discern the various ways of writing a second (or higher) derivative. For example, if $y = f(x)$ is the function being differentiated, then the first derivative can be written variously as $y' = f'(x) = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx} f(x)$. Then its second derivative can be written variously as $y'' = f''(x) = \frac{d^2y}{dx^2} = \frac{d^2f}{dx^2} = \frac{d^2}{dx^2} f(x)$. Third and fourth and higher derivatives' notation follows along those same lines. (And then to confuse the innocent, the names of the variables and functions are changed on you to see if you're paying attention!)

What good are the higher-order derivatives? It's seldom that you'll be asked to look at anything further than a second derivative. And as is written on page 179, one application is the old familiar one concerning velocity and acceleration. If a function gives distance as a function of time, then the first derivative (the rate of change) measures velocity. The second derivative is the rate at which the velocity changes, and that is nothing more than simply acceleration. It's the most famous illustration of first- and second-derivatives, so get familiar with it. It's precisely the realm in which Newton's work was done, as he measured the speed at which planets orbited the Sun.

Added after Day 12 class meeting, September 28, 2009

Today I played catchup, lecturing on the Chain Rule (see above).

Added after Day 13 class meeting, September 30, 2009

Today I continued to play catchup, lecturing on higher-order derivatives (see above). I also began lecturing on Section 2.1, so go to the next chapter's "blog" for my post-lecture notes about that.