Math 1101 – Mathematical Modeling
Crauder, Evans and Noell's *Functions and Change, A Modeling Approach to College Algebra, 3rd ed.*
Lecture Notes for Chapter 3, "Straight Lines and Linear Functions"

You'll find that an important message of this course is that words are important. You will be expected to be able to communicate with sentences. While the correct *number* is of course important, that's not the entire answer. You must supply the proper words that give that number what I call "personality." Without the words, the numbers are just so much garbage.

Notes after Class Meeting #11, Tuesday, September 22, 2009

Well, the rains came yesterday, so the campus was closed today and there was no class. We were slated to begin studying Chapter 3's Section 1, but that will have to be put off until campus reopens and we meet again!

Notes after Class Meeting #12, Thursday, September 24, 2009

Seeing as everyone in the class has had algebra in the past, most of this chapter about linear functions and straight lines should be very easy for everyone. There are many applications for this type of function, since it is the simplest function to define.

Don't forget the simple but important things about straight lines. For instance, "two points determine a straight line." That gets exploited in algebra as the easy way to work out the equation of a straight line. The "slope" of a line is merely the difference in the *y*-coordinates of the two points divided by the difference in the *x*-coordinates of the two points. Remember to keep the order of the coordinates the same as you perform that division. So, for instance, if the two points are (3, 5) and (7, -2) then the slope will be equal to $\frac{5-(-2)}{3-7} = \frac{5+2}{-4} = -\frac{7}{4}$. The phrase "rise over run" is a well-known memory device for making sure that you put the difference of the *y*-coordinates on top for this division!

A different way of asking for that same information about those same two points would be to express the *y*-coordinates using function notation. So, if the function is named *f*, then you might instead be told that $f(3) = 5$ and $f(7) = -2$. But I hope you won't be stumped by that!

The similarity of triangles means that the slope is consistent no matter what part of the straight line is being talked about.

All of that is found in Section 3.1, the "Geometry of Lines." Moving to Section 3.2, "Linear Functions," the author's spell out the familiar equation for a linear function. In the so-called "slope-intercept" form of such an equation, the generic function looks like this: $y = mx + b$. For a specific line, the *m* is replaced by the number for the slope of the line while the *b* is replaced by the number where the line crosses over the *y*-axis. This section has lots of exercises that simply boil down to being given the slope (or the coordinates of two points, from which the slope can be calculated). The slope, together with the coordinates of any point on the line can be used to then work out the value for *b*, the *y*-intercept.

It's important to remember that two lines that are parallel will have the same slope. And an allied piece of information to tuck away is this: two lines that are perpendicular to one another will have slopes that are negative reciprocals of each other. So, for instance, if one of the lines has a slope equal to 4, then the other will have a slope equal to -1/4.

Notes after Class Meeting #13, Tuesday, September 29, 2009

In Section 3.3, "Modeling Data with Linear Functions," your attention is directed towards the basic task of mathematical modeling: coming up with the equation of a straight line when given a table of data. First, it's necessary to assure that the data in the table is appropriately "linear." So the authors direct your attention to a basic indicator that the data in a table is truly linear: both the x values and the y values are evenly spaced. A key example of that is shown in the second table on page 219 of the textbook:

Date	2000	2001	2002	2003	2004	2005
Registered voters	28,321	28,783	29,245	29,707	30,169	30,631

It's easy to ascertain that the top row values are each separated by a difference of 1. A little more arithmetic is needed to check out that the values in the second row (the ones I'll simply call the y values) are each 462 more than the preceding one.

I called attention to the fact that, as we start working with data to graph it and produced equations for linear functions, it's important to use a technique of not doing arithmetic with the actual year value. Instead, the technique of calling the first year value the "base year" and replacing the year values with a number that is the number of years since the base year, will be most handy. Otherwise, the arithmetic needed later gets out of hand. That's why the above table appears at the bottom of page 219 this way:

d	0	1	2	3	4	5
N	28,321	28,783	29,245	29,707	30,169	30,631

On the next page, the authors then use the steps covered in Section 3.2 to produce the equation for a function that perfectly matches the data in this second table: $N = 462d + 28{,}321$. Written properly, with words, a full description of this mathematical model would be:

$N = 462d + 28{,}321$ registered voters in the year that is d years since 2000.

The authors (on page 223) and I talked about using the graphing calculator to load in the data from a table such as this one, together with the mathematical model, and graph them (the data points and the formula) together on one screen. Yep! That line goes right through those data points!

I gave a 10-minute quiz today that was worth 10 points. If you skipped class, you missed out on this scoring opportunity.

Notes after Class Meeting #14, Thursday, October 1, 2009

Although the topic for the day was Section 3.4, "Linear Regression," there was so much time spent on simply working on various homework questions that we almost ran out of time. And although I did introduce the topic of linear regression in the final 10-15 minutes, I completely overlooked handing back the graded quizzes. See the notes for Class Meeting #14 for the details of linear regression.

Notes after Class Meeting #15, Tuesday, October 6, 2009

I did hand back the quizzes today. The class average was 8 out of 10 points.

I provided much more detail and examples about the material in Section 3.4, "Linear Regression," today. Linear regression grows out of the basic human need to try and fit things that are almost in a straight line into a mathematical model that is a straight line. We will not concern ourselves with the details of the formulas that calculate the line with the best slope and *y*-intercept for the line that best matches the data. Rather, we'll use the facilities of the graphing calculator to come up with the line's equation. Suffice to say, it comes down to a type of "averaging" for the coordinates of all the points, even though they're not in a perfectly straight line.

The technique to be used boils down to these steps:

Use the STAT Edit feature to store the coordinates of the data points into the two "list" variables name L1 and L2.

Activate a plot using the STAT Plot feature (it's the second function of the Y= button), making sure that it references the two lists you've used.

Press ZOOM 9 (STATStat) to display the data.

Press STAT and move to the right to the menu named CALC, and choose option #4, LinReg(ax+b). But then be sure to append the characters L1,L2,Y1 before pressing the ENTER key to run the linear regression program, since the program needs to know where the data is that you've stored as well as where in the Y= menu to store the finished regression line's equation.

Then just press GRAPH and you'll see both the data and the regression line graphed together. There should be a very close match in how the points lie versus how the regression line runs.

In Section 3.5, "Systems of Equations," the topic is the familiar one about "solving" a system of two linear functions. That is, we'll seek the single common point between two lines (provided they are not parallel!). The easiest way is to enter both lines' equations into the Y= menu (after you rewrite each one so that the *y* variable is by itself on one side of the equation ... in other words, converting each one into the standard slope-intercept format). The simply use option 5:Intersect from the CALC menu (the 2nd function of the TRACE button), press ENTER three times, and you'll have the coordinates of that common point. I then reviewed the algebraic method, using "substitution," to arrive at the same answer without the need for the calculator (see page 253 of the textbook). For those who are adventuresome and remember about solving the system using the technique called "elimination" or using matrix algebra, the authors provide a review of that in the "Another Look" in the blue-shaded pages beginning on page 254 — but I will not test over that technique.

Thursday this week we'll review for the test over Chapter 3 that you'll have next Tuesday. Do your WebAssign homework, and study the practice test that I've put on this website, then come to class this Thursday with some substantive questions.