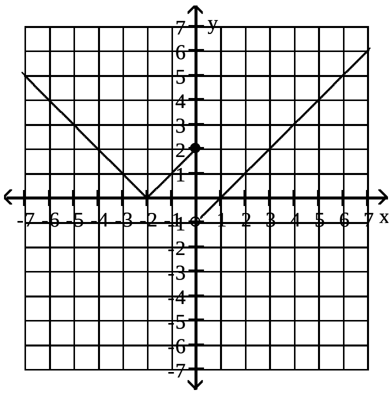
Math 1106 –– Instructor: Bruce Thomas
Fall Semester 2009
Practice Test #1

Please email me ASAP if you find a typographical error or any other kind of error in this version of the practice test.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Decide whether the limit exists. If it exists, find its value.

1) 1) ______



Find $\lim_{x \to 0^-} f(x)$ and $\lim_{x \to 0^+} f(x)$.

A) –2; –1 B) 2; –1 C) 2; 1 D) –1; 2

Notice that these two side limits both involve values of x that are approaching the value of 0, from the left and from the right. The left limit involves the portion of the graph that is to the left of the vertical line $x = 0$. That portion is clearly striving to achieve a height of 2. Thus, the left limit as x approaches 0 from that side is equal to 2. Similarly, the right limit can be easily seen to be equal to –1. Thus, the correct answer choice is **B**.

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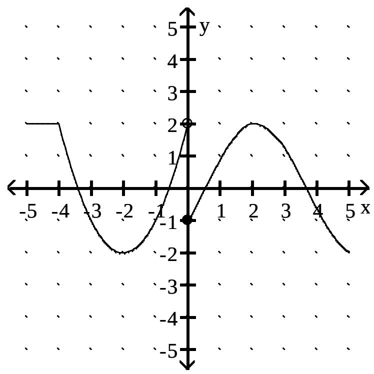
Determine whether the function shown is continuous over the interval (–5, 5).

2) 

2) ______

A) Yes

B) No

It should be very noticeable that the function has a huge vertical gap at $x = 0$, so that the limit does not exist. Because of the 3 mandatory conditions that must be met for a function to be continuous for a particular value of x, this function is not continuous for $x = 0$ because the second mandatory condition requires that there be a limit as x approaches 0. Thus, the correct answer choice is **B**.

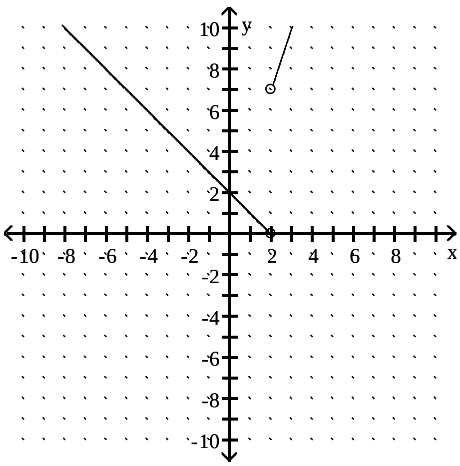
.

Graph the function and then find the specified limit. When necessary, state that the limit does not exist.

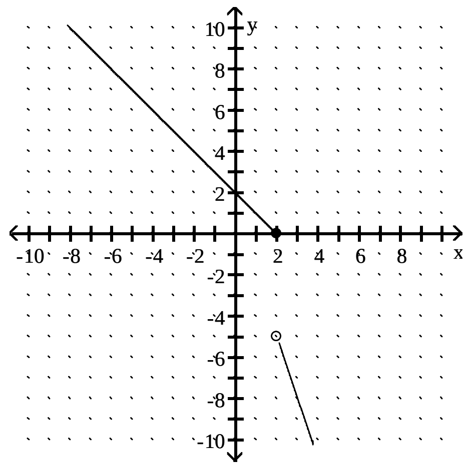
3) $f(x) = \begin{cases} 2 - x, & x \le 2 \\ 1 + 3x, & x > 2 \end{cases}$; $\lim\limits_{x \to 2^+} f(x)$

3) ______

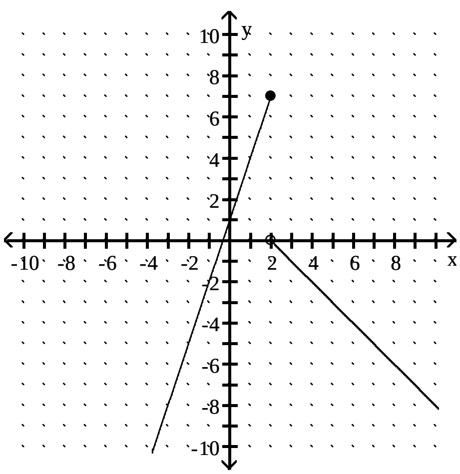
A) $\lim\limits_{x \to 2^+} f(x) = 7$



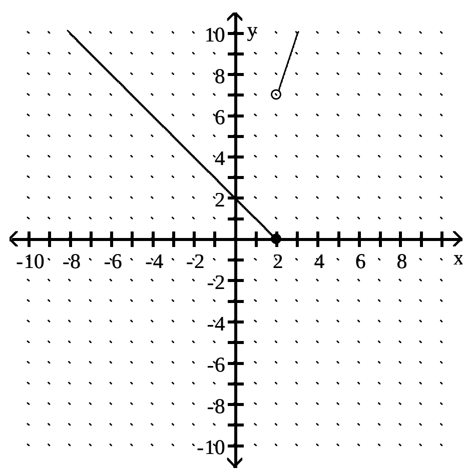
B) $\lim\limits_{x \to 2^+} f(x) = -5$



C) $\lim\limits_{x \to 2^+} f(x) = 0$



D) $\lim\limits_{x \to 2^+} f(x) = 7$



Since it's the right limit at 2 that's being asked about, use the second piece of the definition to determine the right limit at $x = 2$: $1 + 3*2 = 7$. That immediately says that the correct answer must be either (A) or (D). The other two choices are so ridiculously wrong! Check them out, they have very obvious discrepancies in the direction of one or both of the two pieces. But be careful in checking out (A) and (D) -- I though they were identical, but they're not. Look carefully at (A) and see if you can discover the one tiny reason why it is not the right answer. That leaves the correct answer to be **D.**

Evaluate or determine that the limit does not exist for each of the limits (a) $\lim_{x \to 1^-} f(x)$, (b) $\lim_{x \to 1^+} f(x)$, and (c) $\lim_{x \to 1} f(x)$ for the given function f.

4) 4) ______

$$f(x) = \begin{cases} 7x - 10, & \text{for } x < 1, \\ 1, & \text{for } x = 1, \\ -6x + 4, & \text{for } x > 1 \end{cases}$$

A) (a) −2
(b) −3
(c) Does not exist

B) (a) −3
(b) −2
(c) −5

C) (a) −2
(b) −3
(c) −5

D) (a) −3
(b) −2
(c) Does not exist

Plug $x = 1$ into the first piece of the function's defining expressions (7x − 10) to determine that the left limit at 1 is

$$\lim_{x \to 1^-} f(x) = 7 \times 1 - 10 = -3$$
.

Then determine the right limit at 1 by using the third piece (−6x + 4) and plugging $x = 1$ into it. That one turns out to be equal to −2. Since the left and right limits are not the same, that means that "the" limit does not exist at $x = 1$. The correct answer choice is **D**.

Find the limit, if it exists.

5) $\lim_{x \to 2} (x^2 + 8x - 2)$ 5) ______

A) 18
B) Does not exist
C) 0
D) -18

(i) $x^2 + 8x - 2$ is a polynomial expression, representing the implied function being evaluated in this limit question
(ii) all polynomial functions are continuous for every value of x
(iii) by definition, to be continuous for a value of x the limit there has to be equal to the function's value there
(iv) since f(2) = 18, being continuous means the limit there has to be equal to 18 as well.

The correct answer choice is **A**.

6) $\lim_{x \to -8} \frac{x^2 - 64}{x + 8}$ 6) ______

A) Does not exist
B) 1
C) -16
D) -8

Since plugging -8 into the given expression yields the indeterminate result of 0/0, that means there is a limit, but you'll have work to find out what it is. So, in this case use your knowledge of how to correctly factor the numerator portion (since it is the difference of two perfect squares) into its factors. Those factors are $(x - 8)$ and $(x + 8)$. So you get to cancel out the $(x + 8)$ on the top against the one on the bottom. That leaves the simplified expression $(x - 8)$. That's just a straight line function (a polynomial), so its limit is simply what you get when you plug in $x = -8$. Thus the correct answer turns out to be -8 - 8 = -16, which is answer choice **C.**

7) $\lim_{x \to 5} \frac{x^2 + 25}{x + 5}$ 7) ______

A) 5
B) Does not exist
C) 10
D) 0

Pay attention for this one. Start by plugging in $x = 5$. Since the bottom of the rational expression is <u>not</u> equal to zero, then go ahead and evaluate the whole expression with $x = 5$. In this case it turns out to be equal to 5, so the limit is equal to 5 where $x = 5$. The correct answer choice is **A.**

8) $\lim_{x \to 3} \sqrt{x^2 + 4x + 4}$ 8) ______

A) ±5 B) 25

C) Does not exist D) 5

Underneath the radical is a polynomial expression, and polynomial expressions are everywhere continuous and you get their limit by simply plugging in the relevant value for x where the limit is being evaluated. And so, by the rules for limits, the limit of a root of a function that you know the limit for will be the root of that underlying limit. Hence, in this case, just plug in $x = 3$, do the arithmetic, and get the correct answer of 5. The correct answer choice is thus **D.**

Find the limit by using the TABLE and TRACE features of your graphing calculator.

9) $\lim_{x \to 0} \frac{\sqrt{3 + 3x} - \sqrt{3}}{x}$ 9) ______

A) $\frac{\sqrt{3}}{2}$ B) $\frac{1}{2}$ C) 0 D) $\sqrt{3}$

Haul out your graphing calculator and put that rational expression into the Y= menu. Make sure you do it correctly, please! Remember to wrap the numerator portion in a set of parentheses, or else you'll end up with the wrong answer! Press ZOOM 6 and you'll see the function's graph, but it'll be a squished line that hugs the x-axis a lot. To see things better, adjust the picture by pressing ZOOM 2 followed by the ENTER key (to zoom in). Then press the TRACE button. Notice that at the bottom of the screen, there'll be a clear indication that this function is not defined at 0 -- that is, there's a "hole" in the graph there. So how high is the hole? The answer to the question will be the limit. So go just to the left of $x = 0$ by pressing the blue left-cursor key and check out the y-value there; then use the blue right-cursor key and check out the y-value for an x-value that's just a smidgen greater than zero. Those two values (about 0.88 and 0.85) give you a target limit that's somewhere between those two value. So which one of the four answer choices above comes the closest to that. Use the calculator to check out the decimal values of answer choices A and D. It should be clear that the best possibility for the limit is answer choice **A,** since it's roughly 0.87 (while answer choice D is roughly 1.7) and wins the contest, hands down.

Answer the question.

10) Given f(x) = x + 6 and g(x) = x - 6, where is the function f(x)/g(x) continuous? 10) ______

A) The function f(x)/g(x) is continuous for all x except x = -6 and x = 6.
B) The function f(x)/g(x) is continuous for all x.
C) The function f(x)/g(x) is continuous for all x except x = -6.
D) The function f(x)/g(x) is continuous for all x except x = 6.

By using the ratio f(x)/g(x), you've got a rational function. Rational functions are continuous everywhere except for those values of the variable x for which g(x) = 0. In this case, g(x) equals zero only for $x = 6$. So since the rational function has no value where $x = 6$, it fails the official test for continuity at that spot, and only at that spot. Thus the correct answer choice is **D.**

Find a simplified difference quotient for the function.

11) $f(x) = 9x^3$ 11) ______

A) $27x^2$
B) $27x^2 + 27xh + 9h$
C) $27x^2 + 27xh + 9h^2$
D) $27x^2 + h$

Here's the dreaded difference quotient work, carried out to the point where the "h" in the denominator gets annihilated by cancelling it against the factor of "h" that gets developed in the numerator.

$$\frac{f(x+h)-f(x)}{h} = \frac{9(x+h)^3 - 9x^3}{h}$$
$$= \frac{9(x^3+3x^2h+3xh^2+h^3)-9x^3}{h}$$
$$= \frac{(9x^3+27x^2h+27xh^2+9h^3)-9x^3}{h}$$
$$= \frac{h(27x^2+27xh+9h^2)}{h} = 27x^2+27xh+9h^2$$

So the correct answer choice is **C.**

Solve the problem.

12) At the beginning of a trip, the odometer on a car reads 28,312 and the car has a full tank of gas. At the end of the trip the odometer reads 28,604 and there are 2.2 gallons remaining in the tank. The tank can hold a total of 11 gallons. What is the average rate of change of the number of miles with respect to the number of gallons? Assume that the tank was not filled during the trip. 12) _____

A) 292 miles
B) 22.12 miles/gal
C) 33.18 miles/gal
D) 26.55 miles/gal

Everybody knows how to calculate fuel efficiency, right? It's just the number of miles driven divided by the number of gallons burned, okay? But that is easily expressed as a difference quotient:

$$\frac{28{,}604 - 28{,}312}{11 - 2.2} = \frac{292}{8.8} \cong 33.18 \text{ miles per gallon}$$

So the correct answer choice is **C.**

Find the derivative.

13) $f(x) = 5x^{130}$ 13) _____

A) $f'(x) = 650x^{129}$
B) $f'(x) = 650x^{131}$
C) $f'(x) = 650x^{130}$
D) $f'(x) = 5x^{129}$

This is a simple application of the Power Rule for differentiation, in conjunction with the Constant Multiplier Rule:

$$f'(x) = 5 * 130x^{129} = 650x^{129}$$

So the correct answer choice is **A.**

14) $y = 4 - 3x^2$ 14) ______

A) $\frac{dy}{dx} = 4 - 6x$ B) $\frac{dy}{dx} = 4 - 3x$ C) $\frac{dy}{dx} = -6x$ D) $\frac{dy}{dx} = -6$

This is a simple application of all four of the first rules of differentiation that were introduced in Section 1.5 of the textbook:

$$\begin{aligned} \tfrac{d}{dx}(4-3x^2) &= \tfrac{d}{dx}4 - \tfrac{d}{dx}3x^2 \text{ [sum - difference rule]} \\ &= 0 - 3\tfrac{d}{dx}x^2 \text{ [constant and constant multiplier rules]} \\ &= -3 * 2x \text{ [power rule]} \\ &= -6x \end{aligned}$$

So the correct answer choice is **C.**

Find f ′(a) for the given value of a.

15) $f(x) = x^4 + 4x^3 + 2x - 2,\ a = -2$ 15) ______

A) −6 B) 18 C) −8 D) 16

This is just another simple application of the basic rules of differentiation:

$$\begin{aligned} f'(x) &= 4x^3 + 12x^2 + 2 \\ f'(-2) &= 4(-2)^3 + 12(-2)^2 + 2 \\ &= 4(-8) + 12(4) + 2 \\ &= -32 + 48 + 2 \\ &= 18 \end{aligned}$$

So the correct answer choice is **B.**

Find the equation of the line tangent to the graph of the function at the indicated point.

16) $f(x) = x^2 - x$ at $(-3, 12)$ 16) ______

A) $y = -7x - 6$ B) $y = -7x + 6$ C) $y = -7x - 9$ D) $y = -7x + 9$

Since it's known that the tangent line goes through the point $(-3, 12)$ all that's needed is to know its slope in order to write its equation (using the point–slope formula for a straight line).

But since it's a tangent line, its slope must be equal to the slope of the given function when $x = -3$. So, first work out the simple derivative: $f'(x) = 2x - 1$. That means that $f'(-3) = 2(-3) - 1 = -7$.

Now use the point–slope formula: $y - 12 = -7(x - (-3)) = -7x - 21$. That's equivalent to
$y = -7x - 21 + 12$, or simply $y = -7x - 9$.

So the correct answer choice is **C.**

Find all values of x (if any) where the tangent line to the graph of the function is horizontal.

17) $y = x^3 - 3x^2 + 1$ 17) ______

A) 2 B) 0 C) −2, 0, 2 D) 0, 2

This turns out to be really easy. A tangent line that's horizontal has a slope equal to 0. That means you need to solve this equation: $y' = 0$. Using the rules of differentiation leads to the derivative,
$y' = 3x^2 - 6x = 3x(x - 2)$ which equals zero only when $x = 0$ or when $x = 2$. Thus the correct answer choice is **D.**

Find the derivative.

18) $f(x) = 9x^{7/5} - 5x^2 + 10^4$ 18) ______

A) $f'(x) = \frac{63}{5}x^{6/5} - 10x$ B) $f'(x) = \frac{63}{5}x^{2/5} - 10x + 4000$

C) $f'(x) = \frac{63}{5}x^{2/5} - 10x$ D) $f'(x) = \frac{63}{5}x^{6/5} - 10x + 4000$

This is really easy, if you don't mess up your arithmetic with fractions.

$$f'(x) = 9\left(\tfrac{7}{5}\right)x^{2/5} - 5(2)x$$
$$= \tfrac{63}{5}x^{2/5} - 10x$$

So the correct answer choice is **C.**

19) $g(x) = 4x^5 + x^4 - 4x^2 + 7$, find $g'(-1)$ 19) ______

A) 4 B) 28 C) 24 D) 16

Here's another easy question concerning getting the derivative function and plugging in a particular value for x:

$$g'(x) = 4(5)x^4 + 4x^3 - 4(2)x$$
$$= 20x^4 + 4x^3 - 8x$$
$$g'(-1) = 20(-1)^4 + 4(-1)^3 - 8(-1)$$
$$= 20 - 4 + 8$$
$$= 24$$

So the correct answer choice is **C**.

20) $f(x) = \sqrt[5]{x}$ 20) ______

A) $f'(x) = -\frac{6}{5}x^{-6/5}$ B) $f'(x) = \frac{1}{5}x^{-4/5}$

C) $f'(x) = \frac{6}{5}x^{6/5}$ D) $f'(x) = 4(\sqrt[4]{x})$

First, use the rule for fractional exponents to replace the radical:

$$f(x) = \sqrt[5]{x} = x^{1/5}$$

Then use the power rule to get:

$$f'(x) = \frac{1}{5}x^{-4/5}$$

Don't mess up the simple arithmetic of subtracting 1 from 1/5 to get –4/5!!!!!

So the correct answer choice is **B.**

Give an appropriate answer.

21) If $g'(3) = 4$ and $h'(3) = -1$, find $f'(3)$ for $f(x) = 5g(x) - 3h(x) + 2$. 21) ______

A) 17 B) 19 C) 23 D) 25

First, use the Sum–Difference Rule for differentiation on the original function:

$$f(x) = 5g(x) - 3h(x) + 2$$
$$f'(x) = 5g'(x) - 3h'(x)$$

Then just make the simple substitutions given to you for when $x = 3$:

$$\begin{aligned} f'(3) &= 5g'(3) - 3h'(3) \\ &= 5(4) - 3(-1) \\ &= 20 + 3 \\ &= 23 \end{aligned}$$

So the correct answer choice is **C.**

Differentiate.

22) $f(x) = (4x - 6)(5x + 1)$ 22) ______

A) $f'(x) = 40x - 26$
B) $f'(x) = 20x - 26$
C) $f'(x) = 40x - 34$
D) $f'(x) = 40x - 13$

That function is defined as a product of two linear functions. So use the Power Rule:

$$\begin{aligned} f'(x) &= (4x-6)\tfrac{d}{dx}(5x+1) + (5x+1)\tfrac{d}{dx}(4x-6) \\ &= (4x-6)(5) + (5x+1)(4) \\ &= (20x-30) + (20x+4) \\ &= 40x - 26 \end{aligned}$$

So the correct answer choice is **A**.

23) $f(x) = (4x^3 + 9)(5x^7 - 5)$ 23) ______

A) $f'(x) = 200x^9 + 315x^6 - 60x$
B) $f'(x) = 16x^9 + 315x^6 - 60x^2$
C) $f'(x) = 16x^9 + 315x^6 - 60x$
D) $f'(x) = 200x^9 + 315x^6 - 60x^2$

Again, use the Product Rule for differentiation:

$$\begin{aligned} f'(x) &= (4x^3+9)\tfrac{d}{dx}(5x^7-5) + (5x^7-5)\tfrac{d}{dx}(4x^3+9) \\ &= (4x^3+9)(35x^6) + (5x^7-5)(12x^2) \\ &= \left(140x^9+315x^6\right) + \left(60x^9-60x^2\right) \\ &= 200x^9 + 315x^6 - 60x^2 \end{aligned}$$

So the correct answer choice is **D**.

Solve the problem.

24) The profit in dollars from the sale of x thousand compact disc players is $P(x) = x^3 - 3x^2 + 6x + 8$. Find the marginal profit when the value of x is 12. 24) ______

A) \$374
B) \$366
C) \$1404
D) \$1412

Recall that "marginal profit" is business lingo for the derivative of a profit function. It's the rate-of-change of the profit -- i.e., its the current profit that will come from the next batch of 1000 compact disc players. So just differentiate and substitute the current value of x = 12:

$$\begin{aligned} P'(x) &= 3x^2 - 6x + 6 \\ P'(12) &= 3(12^2) - 6(12) + 6 \\ &= 366 \text{ dollars} \end{aligned}$$

So the correct answer is **B.**

25) The median weight, w, of a girl between the ages of 0 and 36 months can be approximated by the function 25) ______

$$w(t) = 0.0006t^3 - 0.0484t^2 + 1.61t + 7.60,$$

where t is measured in months and w is measured in pounds.
For a girl of median weight, find the rate of change of weight with respect to time at age 20 months.

A) 0.882 lb/mo B) 0.394 lb/mo C) 0.086 lb/mo D) 1.362 lb/mo

All this question is asking you to do is to differentiate and substitute $t = 20$.

$$w'(t) = 0.0018t^2 - 0.0968t + 1.61$$
$$w'(20) = 0.0018(20^2) - 0.0968(20) + 1.61$$
$$= 0.394$$

So at 20 months, such a girl of median weight is gaining weight at the rate of 0.394 pounds per month.

The correct answer choice is **B.**

1) B
2) B
3) D
4) D
5) A
6) C
7) A
8) D
9) A
10) D
11) C
12) C
13) A
14) C
15) B
16) C
17) D
18) C
19) C
20) B
21) C
22) A
23) D
24) B
25) B