If you spot any errors in the solutions, email your instructor with an alert, please!!! (See #9)

MATH 1101/24
Practice Test #3
Fall 2009

Instructions: In most of the following questions, if you simply answer with a number or an algebraic expression, that will be insufficient and points will be deducted. As has been told to you in class: A number without any words to describe its meaning is, for the most part, meaningless and worthless! Better yet, you should put the words into complete sentences that appropriately communicate the full meaning of an answer. Write neatly in an organized manner. You must show the calculations that support your answers, even if you do use your calculator; sometimes, drawing and labeling a picture, graph, or diagram can be helpful.

1. (9) You deposit \$75 in a savings account. Here is a table of values of the balance $B = B(t)$ (in dollars), with time t measured in years.

t	0	1	2	3
B	75.00	81.00	87.48	94.48

(a) What is the yearly growth factor for B? (Round your answer to 2 decimal places.)

The t's are spaced in 1-year increments, and $\frac{75.00}{81.00} = \frac{87.48}{81.00} = \frac{94.48}{87.48} = 1.08$, so the yearly growth factor is 1.08 .

(b) Write down the fully-documented exponential function that gives the balance B as a function of time t.

The initial value is 75.00 and the yearly growth factor is 1.08, so the exponential function is defined by these words: $B(t) = 75(1.08)^t$ dollars, where t is measured in years since the initial deposit was made.

(c) What monthly interest rate best represents the growth of this account?

The annual interest rate is 8% (gotten by subtracting 1 from the yearly growth factor and multiplying by 100%). So the monthly interest rate (to 2 decimal places) will be 0.64% (gotten by taking the twelfth-root of 1.08, minus 1).

2. (6) In the early 1990s Microsoft had revenues R that were nearly exponential as a function of time. In the following table, R is revenue in billions of dollars, and t is years since 1989.

t	0	1	2	3	4
R	0.8	1.1	1.8	2.7	3.75

(a) Use exponential regression to find an exponential model for this data. (Be sure to fully document the exponential model that you write down.)

Use the calculator to obtain the exponential regression model. Rounding both initial value and the yearly growth factor to 2 decimal positions, the model is: $R(t) = 0.78(1.49)^t$ billion dollars, where t is the years since 1989.

(b) If this trend had persisted, what would have been Microsoft's revenues for 2001?

Using the rounded exponential regression model shown in part (a) above, with a value of $t = 12$ (because 2001-1989 = 12), the 2001 revenues for Microsoft would come to 86.1 billion dollars; but using the non-rounded version stored in the calculator the answer comes to 94.0 billion dollars. That discrepancy is why it's always best to explain how you arrived at your answer!

3. (3) Suppose the function f is an exponential function with $f(4) = 8$ and $f(5) = 10$. What is the growth factor for f?

Since it's an exponential function, and the difference in the two input values is exactly equal to one, then just obtain the ratio: 10/8 = 1.25 and you've got the growth factor for the exponential function: it's 1.25 .

4. (6) You initially invest \$500 in a savings account that pays a yearly interest rate of 4%. Write a formula for the exponential function giving the balance in your account as a function of the time since your initial investment. Then determine how long it will take for the account's balance to reach \$740.

The yearly interest rate (as a decimal) is 0.04, so the yearly growth factor is one more than that: 1.04 . The initial value is 500, so the function is: $B(t) = 500(1.04)^t$ dollars, t years since the initial investment. Using the crossing graphs method and the calculator with this function, and the number 740, it takes about 10 years to increase to \$740.

5. (6) For the investment described in question #4 above, determine both the monthly interest rate and the quarterly interest rate (express both answers to 3 decimal places).

For the monthly interest rate, follow these steps: the twelfth root of 1.04 is 1.00327374; subtract one and convert that to a percentage gets you the (approximate) monthly interest rate of 0.327% .

For the quarterly interest rate, start by taking the fourth root of 1.04, which is 1.009853407; subtract one and convert that to a percentage to get the (approximate) quarterly interest rate of 0.985%.

6. (6) The following table shows the size, in thousands, of an animal population at the start of the given year. Find an exponential model for this population.

Year	Population (thousands)
2001	2.30
2002	2.51
2003	2.73
2004	2.98
2005	3.25

If you don't use the exponential regression feature, and choose to simply calculate the four ratios, you'll find that they all round (to two decimal places) to a common growth factor of 1.09, and so the exponential model can be written simply as this:

$P(t) = 2.30(1.09)^t$ thousand animals, where t is the number of years since 2001.

But if you instead use the regression feature of your calculator, be sure to correctly specify the base year. If you choose to start your L_1 list with the value of 1, the initial value will be calculated as 2.11, and your words better state that the base year is 2000 and not 2001! Don't put the actual 4-digit years into L_1 — points will be deducted mercilessly!

7. (6) The following table shows the income, measured in thousands of dollars, from sales of a certain magazine at the start of the given year. Find an exponential model for the income.

Year	2001	2002	2003	2004	2005
Income	8.10	8.59	9.10	9.65	10.23

If you don't use the exponential regression feature, and choose to simply calculate the four ratios, you'll find that they all round (to two decimal places) to a common growth factor of 1.06, and so the exponential model can be written simply as this: $P(t) = 8.10(1.06)^t$ thousand dollars of income, where t is the number of years since 2001. But if you instead use the regression feature of your calculator, be sure to correctly specify the base year. If you choose to start your L1 list with the value of 1, the initial value will be calculated as 7.64, and your words better state that the base year is 2000 and not 2001! But woe to the person who mistakenly puts the actual 4-digit years into L_1 — points will be deducted mercilessly!

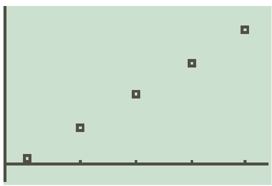
8. (12) One of the two tables below shows data that are better approximated with a linear function, and the other shows data that are better approximated by an exponential function. Determine the appropriate regression to use for each table's data, and write down the regression model for each one.

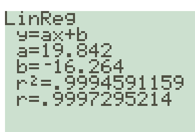
t	*f(t)*
1	3.62
2	23.01
3	44.26
4	62.17
5	83.25

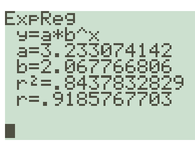
Table A

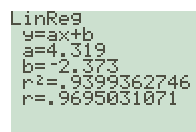
t	*g(t)*
1	3.62
2	5.63
3	8.83
4	13.62
5	21.22

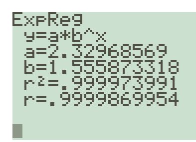
Table B











For Table A, just plotting the points indicates that they are very linear. And with the use of DiagnosticOn, the linear regression r^2 result is much better than for the exponential model. So the better model for Table A is:

$$f(t) = 19.842t - 16.264$$

For Table B, just plotting the points indicates that they are rising exponentially. And with the use of DiagnosticOn, the exponential regression r^2 result is much better than for the linear model. So the better model for Table B is:

$$f(t) = 2.33(1.56)^t$$

9. (12) A pan of cake batter is initially at a room temperature of 75 degrees. The pan is placed in a 350-degree oven to bake. Let $C = C(t)$ denote the temperature of the cake batter t minutes after it is placed in the oven. The temperature of the cake batter is given by the formula C = Limiting value – D, where D represents the difference between the oven temperature and the temperature of the batter.

(a) What is the limiting value of C?

Think about it: C is the temperature of the cake batter. It won't ever exceed the temperature of the oven. And the oven is set to 350 degrees. So the limiting value for C is 350.

(b) It's been found that D is an exponential function. What is the initial value of D?

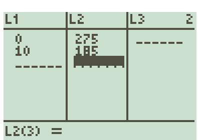
The initial value of C (the cake batter's temperature) is 75 degrees, which is the room temperature. But C is calculated as $C = 350 - D$. So initially, $75 = 350 - D$, which means that the initial value of D is $350 - 75 = 275$.

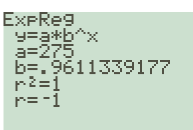
(c) After 10 minutes, the temperature of the cake batter is 165-degrees. What is the formula for D?

Okay, follow along. Part (b) says that D is given by an exponential function. And the initial value of D has been found to be 275. That's when $t = 0$. Now we're told that when $t = 10$, C ~~D~~ has a value of 165. And since C = Limiting value – D, that means $165 = 350 - D$. So solving that simple relationship for D means that when $t = 10$, $D = 350 - 165 = 185$.

What we don't know is the growth factor for D. But if we call that unknown number x, then we can write this relationship now: ~~165~~ $185 = 275x^{10}$. Thus x is equal to the tenth root of the fraction ~~165~~ 185/275 = 0.67 ~~0.6~~ . Using the calculator, and rounding to 2 places after the decimal point, ~~that~~ $(0.67)^{(1/10)}$ comes to ~~0.95~~ 0.96. Therefore, the formula for D (the temperature difference between the cake batter and the oven) will be:
$D = 275(0.96)^t$ ~~$D = 275(0.95)^t$~~ degrees, where t is the number of minutes since the batter was popped into the oven.

Another way to get the exponential function is to notice that since we know two values for D (when $t = 0$ and when $t = 10$), then use the calculator to write the exponential function.





(d) Find a model for the temperature of the cake batter t minutes after it is placed in the oven.

The temperature of the cake batter is given by C. And $C = 350 - D$. So,
$C(t) = 350 - 275(0.96)^t$ ~~$C(t) = 350 - 275(0.95)^t$~~ degrees, where t is the number of minutes since the batter was popped into the oven.

The corrections in red above were made Tuesday afternoon. Thanks for your help in discovering the mistake, Mr. Dodd!

10. (6) A certain phenomenon has an initial value of 11 and grows at a rate of 9% per year. Give a properly annotated exponential function that describes this phenomenon. (Use *t* in years as your variable.)

How about this: $P(t) = 11(1.09)^t$ where *t* is the number of years since the time the phenomenon had a value of 11.

11. (6) Make an exponential model for this data:

x	0	4	8	12
y	1200.0	252.0	52.9	11.1

Be careful with this one. Notice that the *x*'s are increasing by 4 each time. Since the common growth rate (by taking ratios and rounding to 2 decimal positions) is 0.21, you might be tempted to simply write $y = 1200(0.21)^t$ ... but you'd be wrong! If you'd written instead $y = 1200(0.21)^{0.25t}$ you'd be right, or — because that exponential expression can also be written as $y = 1200(0.21^{0.25})^t$ — if you'd known to take the fourth root of 0.21, or simply used your calculator to create the exponential regression to come up with $y = 1200(0.68)^t$, then that's the right answer!

12. (12) You are saving money with the hope of buying a new car after several years. Both the balance in your savings account and the cost of the car grow exponentially over time. Let *t* be time in years since the start of 1998.

(a) The cost *C* (in dollars) of the car at the start of 1998 is $14,000, and the yearly growth factor for the cost is 1.02. Find a formula for *C* as a function of *t*.

Initial value is 14000, yearly growth factor is 1.02: cost of a new car is $C(t) = 14000(1.02)^t$ dollars, *t* years after 1998.

(b) At the start of 1998 you invest $12,000 in a savings account. The yearly percentage growth rate for the account balance *B* (in dollars) is 4.3%.

i. Find the yearly growth factor for *B*.

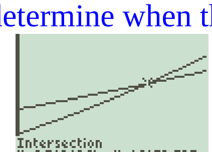
The yearly growth factor is just the yearly percentage growth rate (as a decimal) plus 1: 1 + 0.043 = 1.043

ii. Find a formula for *B* as a function of *t*.

Initial value is 12000, yearly growth factor is 1.043: account will be worth $B(t) = 12000(1.043)^t$ dollars, *t* years after 1998.

iii. Use your answer to Part (a) and to Part (ii) above to determine at what time your account balance will be large enough so that you are able to afford the new car.

You want to know the value of *t* when the two amounts will be equal. So put both functions into your calculator, and use the crossing graphs method to determine when the two graphs intersect. It turns out to be about 7 years later, or roughly 1998 + 7 = 2005.



13. (10) The following table gives the Dow Jones Industrial Average *D* as a function of time *t*, measured in years since 1987. (Assume that a whole number value for *t* indicates the beginning of that year.)

t	0	4.75	7.9	8.6	9.85
D	2000	3000	4000	5000	6000

(a) Use exponential regression to find an exponential model for *D*.

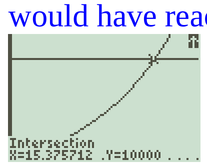
Plugging the data into the calculator and running an exponential regression, the exponential model (with values rounded to 2 decimal places) is: $D(t) = 1906.95(1.11)^t$ points for the Dow Jones Industrial Average *t* years since 1987.

(b) What is the annual percentage growth rate of *D*?

Since the annual growth factor is 1.11, subtracting 1 from that and converting to a percentage gives an annual percentage growth rate of 11% for the Dow Jones Industrial Average.

(c) Based on the exponential model of *D*, when do you predict that the Dow-Jones will reach 10,000?

Using the unrounded version of the exponential regression obtained to answer part (a) above, the crossing graphs method indicates that the DJIA would have reached the 10,000 point level about $15\frac{1}{3}$ years after 1987, or in the spring of 2002.



Using only the rounded model, the intersection point would have been achieved just shortly later, but still in 2002. (For those who are interested in finance, you might like to know that the DJIA actually first reached the 10,000 point level somewhat earlier, in the Spring of 1999. And after all the economic misery of the first few years of the 21st century, when the DJIA took a nosedive after 2001, it has only climbed back up to the 10,000 point level in the last 2-3 weeks. This week, it's back down to 9,712 points. Drat!)