Please let your instructor know if you find some kind of typographical error in this document.

MATH 1101/24
Practice Test #2
Fall 2009

Instructions: In most of the following questions, if you simply answer with a number or an algebraic expression, that will be insufficient and points will be deducted. As has been told to you in class:
A number without any words to describe its meaning is, for the most part, meaningless and worthless!
Better yet, you should put the words into complete sentences that appropriately communicate the full meaning of an answer. Write neatly in an organized manner. You must show the calculations that support your answers; drawing and labeling a picture can be helpful.

1. (6) You are standing at the center of a circular circus tent, where the height is 26 feet. The slope of the tent's roof is -0.6. If you walk 10 feet straight toward the outer wall of the tent and stop, how high is the roof of the tent at that spot?

> The slope of -0.6 means that the height decreases by 0.6 feet for every foot away from the center you walk. Thus, walking 10 feet away from the center means that the roof's height decreases by 10*0.6 = 6 feet. So at this point, the roof is 26 – 6 = 20 feet tall.

2. (6) To conform with the requirements of the Americans with Disabilities Act (ADA), a building which has steps up to its front door (the land in front of the building is perfectly flat) also has a straight ramp that permits easy access to the building for those confined to wheelchairs. The slope of the wheelchair ramp is 1/12. If the steps up to the front door are 2 feet high, how far away from the building is the start of the ramp?

> A slope of 1/12 means that the ramp increases 1 foot in height for every 12 feet of horizontal span. So, in order to rise 2 feet to reach the top of the steps, it must start 2 * 12 = 24 feet away from the building.

3. (6) Suppose that *f* is a linear function such that *f*(3) = 5 and *f*(7) = -4. Find the equation for *f* (in slope-intercept form).

> The information give says that the line goes through the two points referred to by the two order pairs (3, 5) and (7, -4).
>
> Calculate the slope ("rise over run"): $\frac{-4-5}{7-3} = \frac{-9}{4}$. So, using the point-slope form of the equation for a straight line, write $y-5 = \frac{-9}{4}(x-3)$. In slope-intercept form, this becomes $y = \frac{-9}{4}x + \frac{47}{4}$.

4. (8) One rule of thumb that relates weight to height for adult males says that a man who is 1 inch taller than another is expected to be heavier by 5 pounds. Another rule of thumb says that a man who is 70 inches tall should weight 170 pounds. On the basis of these two rules of thumb, state a formula that expresses weight as a linear function of height. Be sure to identify the meaning of the letters you use.

Since the function is linear, notice that the slope works out to be 5 pounds per inch of height. Letting W stand the weight and H stand for the height, a slope of 5 means that the function works out to be W = 5H + b. Using the second rule of thumb (that says "when H = 70 then W = 170"), the formula gives this: 170 = 5*70 + b. Solving that for "b", it turns out that b = -180. Therefore the formula is:

W = 5H – 180 pounds, where H is measured in inches.

5. (9) One of these tables has data that is linear:

Table A

t	0	2	4	6	8	10
$f(t)$	6.7	7.77	9.02	10.46	12.13	14.07

Table B

t	0	2	4	6	8	10
$g(t)$	5.8	7.53	9.26	10.99	12.72	14.45

(a) Is the data in Table A linear? Explain why it is or isn't.

No, the data in Table A is not linear. Notice that although the values in the first row are each 2 more than the previous one, the values in the second row vary wildly: 1.07, then 1.25, then 1.44, then 1.67, and finally 1.94. So the data is certainly not linear.

(b) Is the data in Table B linear? Explain why it is or isn't.

Yes, the data in Table B is perfectly linear. Notice that the values in the first row are each 2 more than the previous one. The values in the second row are similarly consistently space, with each one being exactly 1.73 more than the previous one.

(c) For the data that is linear, find a formula for the linear function.

Using the data from Table B, which is linear, the slope can be calculated by just using the first two pairs of data: $\frac{7.52-5.8}{2-0}=\frac{1.73}{2}=0.865$. Then, in point-slope form, $g(t)-5.8=0.865(t-0)$ and this converts to $g(t)=0.865t+5.8$.

6. (6) For the following table, write the linear model that matches the data:

x	1	4	7	10
y	2	-4	-10	-16

This data is perfectly linear (since both the x's and the y's are each evenly space). The slope can be calculated by just using the first two pairs of data: $\frac{-4-2}{4-1} = \frac{-6}{3} = -2$. Then, in point-slope form, $y-2=-2(x-1)$ and this converts to $y=-2x+4$.

7. (9) The following table shows the number, in millions, graduating from high school in the United States in the given year.

Year	Number graduating (in millions)
1995	2.59
1997	2.75
1999	2.91
2001	3.07

a. Find a formula for a linear function that models these data.

This data is perfectly linear (since both the years and the number of graduates are each evenly space). The slope can be calculated by just using the first two pairs of data: $\frac{2.75-2.59}{2-0} = \frac{0.16}{2} = 0.08$. Then, in point-slope form, $G-2.75=0.08(Y-2)$ and this converts to $G=0.08Y+2.59$ million graduates, Y years after 1995. (Notice: I chose to represent the actual year by a number that is the number of years since 1995.)

b. Specify the slope of this model, and explain in practical terms the meaning of that number.

The slope value of 0.08 means that the number of graduates is increasing by 0.08 million per year.

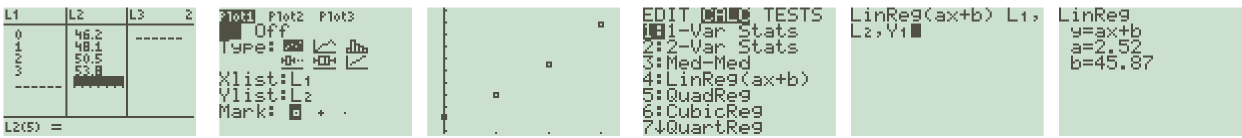
c. How many 2004 high school graduates will there be according to this model?

The year 2004 is 9 years after 1995. Therefore, the number of graduates is represented, using functional notation, as G(9). Plugging Y = 9 into the function's definition yields this answer: G(9) = 0.08*9 + 2.59 = 0.72 + 2.59 = 3.31 million graduates in the year 2004.

8. (8) The following table gives the percentage of American homes with cable TV. Find the equation of a linear regression model, and write the model with appropriate descriptors.

Year	Percent of homes with cable TV
1985	46.2
1986	48.1
1987	50.5
1988	53.8

This data is not perfectly linear (although the years are evenly spaced apart, the percent of homes with cable TV fluctuate erratically). But plug this into the calculator and let it produce a linear regression formula that "fits" the data to a linear line through linear regression.



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The linear regression formula is therefore P = 2.52Y + 45.87 percent of homes with cable TV, where Y is the number of years since 1985. Notice that I chose to specify the year in terms of the number of years since the base year of 1985.

9. (12) Solve the following linear system:

$$\begin{aligned} -6.6x &- 26.5y = 17.1 \\ 6.9x &+ 5.5y = 8.4 \end{aligned}$$

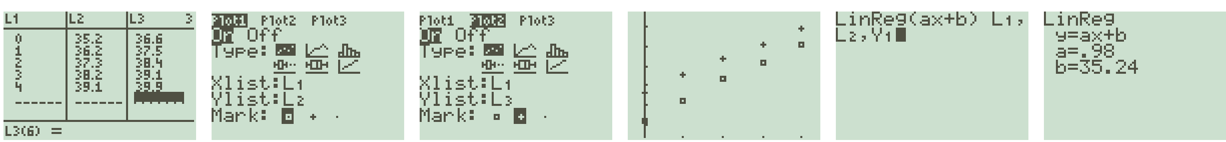
Rewrite the second linear function by solving it for y: $y = \dfrac{-6.9x + 8.4}{5.5}$. Plug this into the first linear function: $-6.6x - 26.5 * \left(\dfrac{-6.9x + 8.4}{5.5}\right) = 17.1$. Multiplying things out results in this: $-6.6x - \left(\dfrac{-182.85x + 222.6}{5.5}\right) = 17.1$. This continues to simplify to: $-\dfrac{726}{110}x + \dfrac{3657}{110}x - \dfrac{4452}{110} = \dfrac{1881}{110}$ or (by combining like terms and multiplying through by 110) $2931x - 4452 = 1881$ and so $x = \dfrac{2111}{977}$. Plugging this back into the expression for y yields $y = \dfrac{-5781}{4885}$.

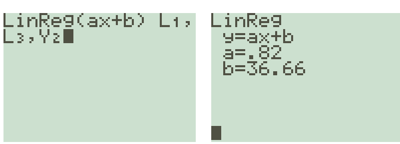
10. (12) The table below shows the average salaries, in thousands of dollars, of elementary and secondary classroom teachers in public schools in the given years.

Year	Elementary	Secondary
1994	35.2	36.6
1995	36.2	37.5
1996	37.3	38.4
1997	38.2	39.1
1998	39.1	39.9

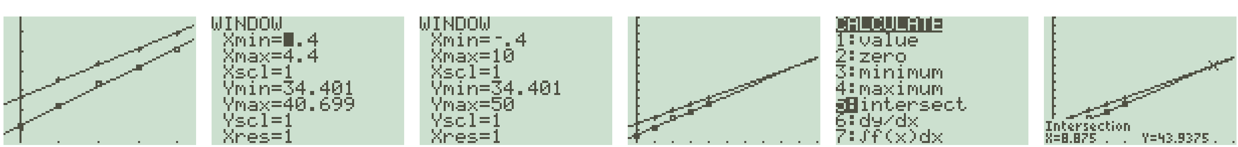
a. Using your calculator, create a linear regression model for the salaries of elementary classroom teachers as a function of time, and another linear regression model for the salaries of secondary classroom teachers as a function of time. Write down both linear models, correctly annotated with words that clearly and unambiguously explain the meanings of the variable names you have chosen to use.

This data is not perfectly linear (although the years are evenly spaced apart, both the elementary and secondary salaries fluctuate in their differences). But plug all this data into the calculator and let it produce two linear regression formulas that "fit" the data to linear lines. Notice that I have used 1994 as the base year. From the two regressions (shown below), the formula for the salaries of elementary classroom teachers is modeled by the linear function $E(t) = 0.98t + 35.24$ thousand dollars, where t is the number of years since 1994. The formula for the salaries of secondary classroom teachers is modeled by the linear function $S(t) = 0.82t + 36.66$ thousand dollars, where t is the number of years since 1994.





b. On the basis of the two regression lines, and assuming that the trends continue, in what year will the average salary of elementary classroom teachers in public schools be the same as that for secondary classroom teachers?



Using the calculator, the two linear models will intersect at the point that is 8.875 years after 1994, or about 2003. (And at that time, both types of teachers will be earning about 43,937.50 per year.)

11. (18) The number of tourists visiting a certain country is a *linear* function of time.

(a) Using this fact, fill in the blanks in the following table. Be sure to show your work.

Date	1979	1983	1987	1991	1995
Number of tourists (in thousands)	103	93.4	83.8		

The years are spaced evenly apart; also, those 3 sets of figures are spaced evenly apart (decreasing by 9.6 thousand tourists each 4 year span). So an exact linear function is appropriate. So, for 1991 it should have 83.8 – 9.6 = 74.2; and for 1995 it will be 74.2 – 9.6 = 66.6 .

(b) Find a formula for a linear function that models the data in the table. (You need to choose variable and function names. Be sure to state what the units are.)

The slope is -9.6/4 = -2.4 thousand per year. So using the data for 1979 (and using 1979 as the base year) the function can be written as $N(t) = -2.4t + 103$ thousand tourists, t years since 1979.

(c) Explain in practical terms the meaning of the slope of the function you found in Part (b).

The slope just means that the number of tourists is decreasing by 2.4 thousand per year.

(d) Use your formula from Part (b) to predict the date when the number of tourists will drop to 57.4 thousand.

All that's needed is to solve the equation 57.4 = -2.4t + 103. That means -45. = -2.4t and so t = 18.75 . Therefore, the number of tourists will be down to 57,400 per year 18.75 years after 1979, or about 1998.