

MATH 1101/24 — Mathematical Modeling — Fall 2009 — Instructor: Bruce Thomas
Practice Test #1

Instructions: Round answers to 2 decimal places, but if the context of the question clearly calls for an answer that could only be a whole number, then round up to the next whole number. In most of the following questions, if you simply answer with a number or an algebraic expression, that will be insufficient and points will be deducted. As has been told to you in class: A number without any words to describe its meaning is, for the most part, meaningless and worthless! Similarly, algebraic expressions floating by themselves are invalid; they should be part of an equation, with words appended that completely explain the meaning of variable names, with appropriate units of measurement appended. Use words with the numbers to communicate the full meaning of an answer. You must show the calculations that support your answers. The point-value for each question is shown in parentheses after the question number.

1. (6) It has been determined that the size of an animal population is described by the function $N(t) = 0.5 + 2t \times 2^{-t}$, where t is measured in years and N is measured in thousands of animals.

(a) Calculate $N(4.5)$ and explain its meaning.

(b) What is the initial population size?

(c) Use functional notation to express the population size after 18 months, and then calculate that population size.

2. (6) The following is a partial table of values for $f = f(x)$.

x	0	3	6
$f = f(x)$	50	55	61

(a) Find the average rate of change for f between $x = 0$ and $x = 3$.

(b) Find the average rate of change for f between $x = 3$ and $x = 6$.

(c) Use your answer to part (b) to estimate the value of $f(4)$.

3. (4) Suppose that f is a linear function such that $f(3) = 5$ and $f(7) = -4$. Use this information to write the equation for f in the slope-intercept form of an equation. (Leave the slope and y-intercept info as fractions.)

4. (18) A trainee is hired to test personal computers as they come off the assembly line. The learning curve for an average trainee is given by

$$N = \frac{200}{4 + 21e^{-0.1t}},$$

where N is the number of computers tested per day, and t is days on the job. (**Hint:** *If you have correctly entered this into your calculator, then a value of 5 for the t will result in a value of about 11.95 for N .*)

- (a) Make a table showing the number of computers tested per day for 0, 10, 20, and 30 days on the job.

- (b) Use functional notation to express the number of computers tested per day after 25 days on the job and then calculate its value.

- (c) What is the average rate of change in the learning curve from 20 to 25 days? From 25 to 30 days?

- (d) Explain what your answers in Part (c) tell you about the growth in the number of computers tested per day.

- (e) What is the limiting value for this function? Explain how you know.

- (f) Write a sentence that interprets the meaning of the limiting value in part (e) above.

5. (8) The weekly cost C (in dollars) of running a telemarketing firm is related to the number N of telephone operators. There is a fixed cost of \$1500 per week, and each telephone operator costs the firm \$300 per week. For example, if there are 10 operators, then the weekly cost is \$4500.

(a) Use a formula to express the weekly cost C as a function of N , the number of telephone operators.

(b) Explain in practical terms what $C(8)$ means, and then use your formula from Part (a) to calculate it.

(c) Use functional notation to express the weekly cost if 5 telephone operators are used, and then use your formula from Part (a) to calculate that cost.

(d) The costs of running the firm have now increased, and the new weekly cost D (in dollars) is given by the formula $D = 1550 + 350N$, where N is the number of telephone operators. How much does each telephone operator cost the firm now?

6. (4) You pay \$500 to rent a special area in a restaurant. In addition, you pay \$10 for each guest. Write a formula that gives your total cost C , in dollars, as a function of the number n of dinner guests.

7. (8) Banks and other lending institutions typically require that their loans be paid back in monthly installments all of the same size. The amount of the monthly loan payment, M , depends on three things: the amount that was borrowed, P ; the monthly interest rate (expressed as a decimal), r ; and the number of months over which the loan will be totally repaid, m . The formula to calculate the monthly loan payment is

$$M = \frac{P \times r \times (1+r)^m}{(1+r)^m - 1} .$$

Use functional notation to express the monthly loan payment for each the following loans, and then use the given formula to calculate the amount of the monthly loan payment:

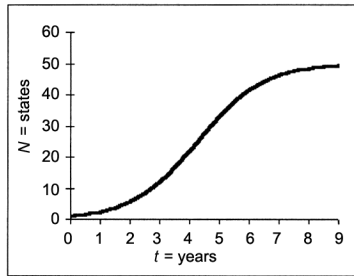
- (a) A car was purchased for \$22,495 and the entire amount was borrowed from a bank at a monthly interest rate of 0.75%, with monthly payments extending for a full 5 years.

- (b) A young couple is about to purchase their first house. The agreed upon purchase price is \$245,495 and they plan to make a down payment equal to 10% of the purchase price, and borrow the remainder of the purchase price. Their lender has agreed to lend them what they'll need to borrow at an annual percentage rate of 6%, with monthly payments over a 30 year period.

8. (8) The person who purchased the car in part (a) of question (7) above is curious to see how much his monthly payments would have been if he had decided on a different length of time for the loan. Create a vertical table below that shows the monthly loan payment amount for various lengths of time, starting with 3 years and ending with 5 years, with increments of 6 months between each table entry.

9. (4) Enhance the table you created in question (8) so that it also shows the total amount that will be paid to the lender over the full length of the loan in each case.

10. (18) In recent years many states have passed mandatory seat belt laws. The total number N of states that had a seat belt law in effect t years after 1984 is given by the graph below.



- (a) How many states had a seat belt law in effect in 1989?
- (b) In what year were there seat belt laws in effect in 40 states?
- (c) When was the graph concave up? Explain how the number of states with seat belt laws was changing during this period.
- (d) When was the graph concave down? Explain how the number of states with seat belt laws was changing during this period.
- (e) Between which two years was the time of fastest growth? What name is given in mathematics to the spot on a graph where the rate of change is the greatest?
- (f) Does the function represented by this graph have a limiting value? What is it? Explain the reasoning you applied to arrive at your answer.

11. (16) A company buys a new color photocopy machine for \$6,180. The machine decreases in value (depreciates) by \$515 per year. The value V of the machine is a function of time t in years since the machine was purchased.

(a) Make a table of values to show the value of the machine for 0 to 4 years.

(b) Use a formula to express the value V as a function of t .

(c) How long will it take for the machine to lose half its original value?

(d) How long will it take for the machine to depreciate to the point where it has no residual value?