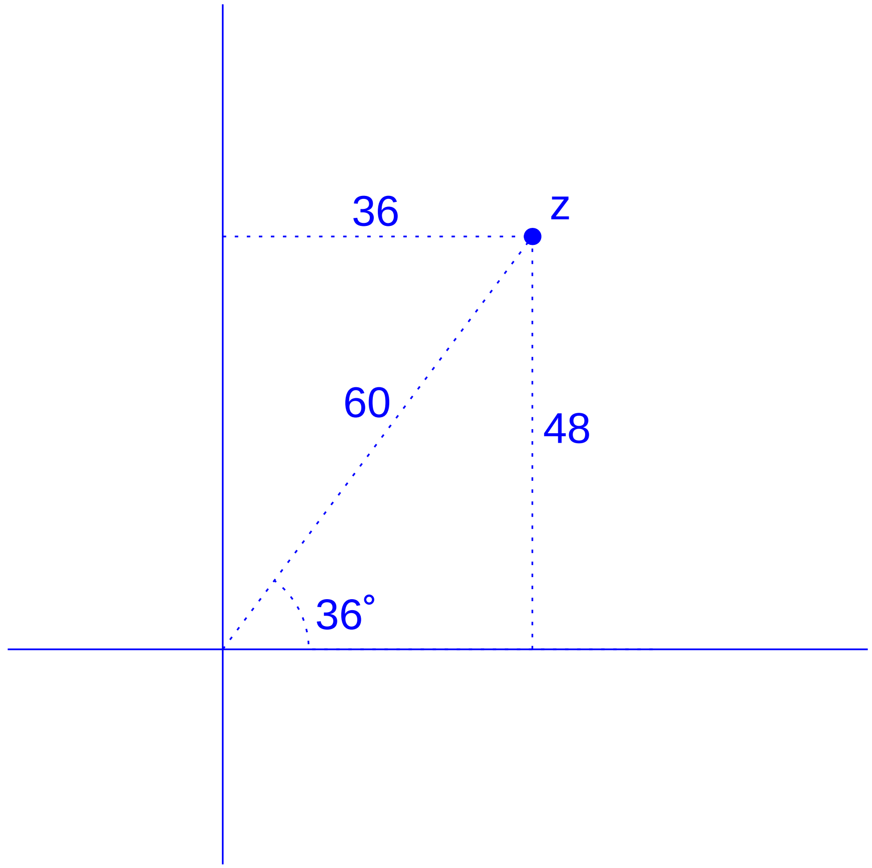
CSIS 4490/02—Lecture 2c

Subset of Lecture 2

Phasors

Graphing Complex Numbers

The set of all complex numbers forms a plane.



Polar Coordinates

- Using r and θ to specify points in the plane.
- Cartesian to polar: $r = \sqrt{x^2 + y^2}$ and $\theta = \arctan(y/x)$
- Polar to cartesian: $x = r\cos(\theta)$ and $y = r\sin(\theta)$

Complex Exponentials

- Euler's formula

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

- With amplitude:

$$re^{j\theta} = r\cos(\theta) + jr\sin(\theta)$$

- $re^{j\theta}$ represents all complex numbers.

- r is the *amplitude* and θ is the *phase*.

Manipulating Complex Exponentials

- $$r_1 e^{j\theta_1} \cdot r_2 e^{j\theta_2} = (r_1 r_2) e^{j(\theta_1 + \theta_2)}$$
- Multiplying two complex numbers multiplies the amplitudes and adds the phases.

Complex Exponential Signals

- $Ae^{\omega_0 t+\phi}$ is a complex exponential signal.

- A sinusoidal signal is the real part of a complex exponential signal:

$$A\cos(\omega_0 t + \phi) = \Re(Ae^{j(\omega_0 t+\phi)})$$

- A sum of sinusoidal signals can be manipulated by manipulating the corresponding sum of complex exponetial signals.

Adding Phasors

Adding two phasors of the same frequency results in a phasor of that common frequency.

$$
\begin{aligned}
& 5e^{j(20\pi t+\frac{\pi}{4})} + 6e^{j(20\pi t+\frac{\pi}{6})} \\
& \quad = e^{j(20\pi t)}(5e^{j\frac{\pi}{4}} + 6e^{j\frac{\pi}{6}}) \\
& \quad \approx e^{j(20\pi t)}(8.7 + 6.5j) \\
& \quad \approx e^{j(20\pi t)}10.9e^{j\frac{\pi}{5}} \\
& \quad = 10.9e^{j(20\pi t+\frac{\pi}{5})}
\end{aligned}
$$

Adding cosines

Taking the real part of the formulae in the previous slide we get a formula relating sinusoids.

$$5\cos(20\pi t + \frac{\pi}{4}) + 6\cos(20\pi t + \frac{\pi}{6}) \approx 10.9\cos(20\pi t + \frac{\pi}{5})$$

This would be difficult to derive simply from the standard trigonometric identities. Using complex exponentials simplifies many computational problems.

Using MatLab

We'll use MatLab to do a comparison of the cosine signals from the last example.

- First we'll graph both the cosine sum and it's simplified version.
- Then we'll subtract the samples and look at the largest difference to estimate how far off we are.

MatLab Basics

- Interactive tool for vector and matrix calculations.
- How to use help
- Basic functions and operators
- Calculates with complex numbers
- ; suppresses display of calculation result

MatLab Vectors

- Array of numbers, indexed starting at one.
- Matrices will be introduced later, but the vector feature is not as general as with C++ or Java.
- Creating vectors
  - `[<start> : <step> : <end>]`
  - `linspace(start,end,numberOfSamples)`

Vector Operations

- Element-wise operations: +   -   .*   ./

- Scalar-Vector operations are carried out by creating a vector by repeating the scalar and then using an element-wise operation.

- True matrix operations are available, we'll discuss them later.