PHYSICS

Snell's Law

Introduction

Whenever a wave encounters an interface in which its speed changes (an impedance change), it will do two things: reflect and refract. Reflection is a process that most of us understand very well. If you have ever looked at yourself in a mirror or heard an echo from a distant wall, you have experienced this property. The principles governing reflection are quite simple. The angle at which the wave hits the surface is equal to the angle at which it reflects, albeit on the opposite side of the normal to the surface. If the surface is smooth, then we have specular reflection in which the entire wave all reflects at this angle. If it is non-smooth and random, then different parts of the wave reflect at randomly different angles, thus distorting the wave.

Refraction is a somewhat less obvious attribute of a wave encountering an impedance mismatch. Refraction occurs when the wave is able to travel through the interface into the other medium. If the wave comes in at a zero incidence angle, then it refracts at zero angle. However, if it comes in at some angle, then it will transmit into the second medium at an angle that depends greatly upon the speed of the wave in each medium. We rarely hear such an effect with sound, other than if we have some sort of unusual temperature profile in the air, such as is found above a warm pond on a cold day. With light, we might have noticed the effects of refraction if we have ever looked into a pond or pool at an angle, or if we look at fish in an aquarium. In both of these incidences, we have probably noticed that objects in the water have a strange appearance and might not be where they appear to be.

Theory

The rule that governs how waves are refracted as they pass between media is known as Snell's law. It is a very simple relationship, involving nothing more than sine functions

$$n_i \sin \theta_i = n_t \sin \theta_t$$

where n is the index of refraction, θ is the angle of the light ray, the i subscripts refer to the incident medium, and the t subscripts refer to the medium into which the light was refracted. As this relationship shows, if the index of refraction of the transmitted medium (n_t) is larger than the index of refraction of the incident medium (n_i), then the transmitted angle (θ_t) will be smaller than the incident angle (θ_I).

What it also shows is that if the inverse ($n_i > n_t$) is true, then the transmitted angle will be larger than the incident up to a point. Since the transmitted angle can never go beyond 90 degrees, this means that there is a limit for incident angles under these conditions beyond which no light is transmitted. This is known as the Brewster angle, and it occurs when the transmitted angle is 90 degrees, or

$$\theta_I = \sin^{-1}\left(\frac{n_t}{n_i}\right)$$

For angles greater than this incident angle, we have total internal reflection. This is the basis for such devices as fiber optic cable that seems to “trap” light as it moves down the cable.

Procedure

To study Snell Law, we will investigate light passing through two different apparatii: a semicircular acrylic solid and a parallel-sided acrylic solid. In the first part of this experiment, we will change the incident angle of a light beam on the middle of the flat face of the semicircular solid. The beam will refract inside of the acrylic, moving at a new angle. However, as the backside of the solid is circular, the beam leaving it will be normal to the surface, and will not refract at a new angle after it has re-entered the air. Thus, the angle of refraction for traveling through the acrylic solid can be measured from the beam exiting the backside of it, as there will be no difference.

Fig. 1: Experimental set up for first part

1. Set-up the equipment as in Figure 1. Before placing the acrylic semicircle on the table, position the light source so that the light beam is traveling down the center of the table, i.e. it passes through both 90^o markings with the table is set as in Figure 2 (Note: you will need the lights in the room turned off for this experiment). Place the acrylic semicircle so that it is aligned with the center line and has its midpoint at the center of the table (Figure 2).
2. Turn the optical table until the beam strikes the surface at a 5 degree angle, i.e. turn the optical table 5 degrees in either direction.
3. Measure the angle at which the beam leaves the optical table by noting the angle at which the refracted light ray strikes the optical table. Record this on the activity sheet.
4. Turn the optical table another 5 degrees and repeat the measurement.
5. Continue to do this until you reach an incidence angle of 60 degrees.
6. Plot sin θ_i vs. sin θ_t as well as calculate the average index of refraction. Compare this to the theoretical value.

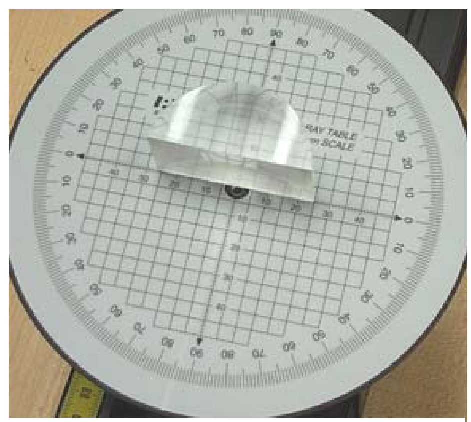

Fig. 2: Acrylic semicircle on table

In the second part of the experiment, we will look at multiple reflections from a slab of transparent material. When an incident light ray strikes a slab of material at an angle other than perpendicular, the beam is reflected multiple times internally in the slab (see Figure 3). Using Snell's Law and a little geometry, we know that

$\sin \theta_i = n_t \sin \theta_t$, and

$x = 2d \tan \theta_t$

Using similar triangles, we also know that

$s = x \cos \theta_i$

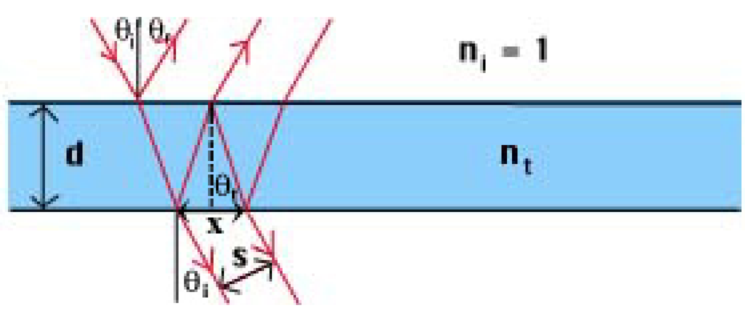

Fig. 3: Diagram of reflections and refractions in a slab

If we solve this equation for x and plug this into the second of the equations above, we get

s sin θ_i = 2 d tan θ_t

The tangent function is equal to the sine over the cosine. Using the fact that sin θ_i = n_t sin θ_t and that 1 = $\sin^2 \theta + \cos^2 \theta$, we substitute into this equation to get

$$s = \frac{2d \sin \theta_i \cos \theta_i}{\sqrt{n_t^2 - \sin^2 \theta_i}}$$

By measuring d and s, one can calculate the index of refraction of the material.

1. Remove the semicircular acrylic slab and replace it with the trapezoidal acrylic slab. Position it such that the slab face is normal to the beam while the beam intersects the optical table at 0 degrees. The object should be placed as in Figure 4 such that refracted beam will reflect off of the parallel back surface.
2. Move the table 10 degrees and measure the separation s for two transmitted beams. This should be done using the millimeter scale on a ruler held perpendicular to both reflected beams. Record your measurement on the activity sheet.
3. Repeat this for the optical table placement at 20^o and 30^o.
4. Turn off the light source.
5. Measure the thickness of the slab d.
6. Using the formula above, calculate the index of refraction for glass for each run. Average these values and compare them to the theoretical answer.
7. Answer the questions on the activity sheet.

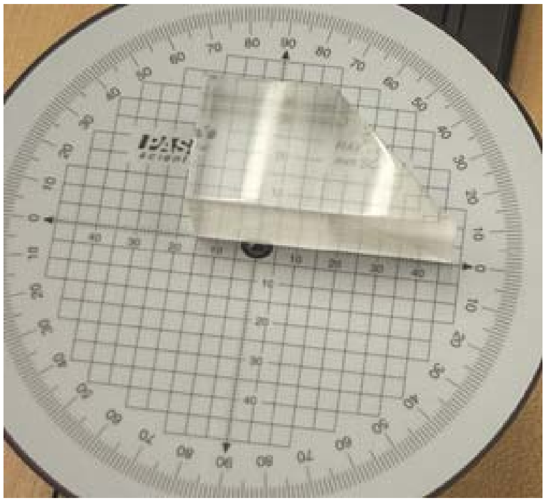

Fig. 4: Trapezoidal acrylic object aligned with ruler on optical table

Physics Activities	Activity Sheet Snell's Law

Name:

Part One

Incident Angle	5^o	10^o	15^o	20^o	25^o	30^o	35^o	40^o	45^o	50^o	55^o	60^o	Avg.
Refracted Angle													
Index of refraction													

1. How close is your measured value for the index of refraction to the theoretical value of 1.5? Are there any random or systematic errors that could account for these differences?

Part Two

Incident Angle	s	d	Index of Refraction
10^o			
20^o			
30^o			
		Average	

2. How close is your measured value for the index of refraction to the theoretical value of 1.5? Are there any random or systematic errors that could account for these differences?